

MONTHLY NOTICES  
OF THE  
ROYAL ASTRONOMICAL SOCIETY

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MEETING OF 1935 APRIL 12

Mr. J. H. Reynolds, President, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Mrs. Frances Draper, 4 Whitehall Court, London, S.W. 1 (proposed by R. Inwards) ;

Y. Iba, Ohte, Kobe, Japan (proposed by H. Shapley) ;

Inigo Jones, Director of the Cromanshurst Solar Observatory, Queensland, Australia (proposed by R. B. Bousfield) ;

Reginald Victor Jones, D.Phil., University Observatory, Oxford (proposed by H. H. Plaskett) ;

John Pickston, B.Sc., 15 Stanley Street, Brighouse, Yorks (proposed by W. Porthouse) ;

Charles Frederick Ortmann Smith, 22 Inverleith Gardens, Edinburgh (proposed by J. McD. Field) ;

Henry John Spooner, M.I.Mech.E., A.M.Inst.C.E., F.G.S., 5 St. George's Terrace, Regent's Park, London, N.W. 1 (proposed by R. Inwards) ; and

Mrs. Ruby Ta' Bois, Winfield, 33 High Road, Woodford Green, Essex (proposed by A. P. Freeman).

The following candidate was proposed for election as Fellow of the Society, the name of the proposer from personal knowledge being appended :—

Reginald Douglas Kreltshheim, B.Sc., F.R.Met.Soc., Junior Technical Assistant, Colombo Observatory, Ceylon, and 72 Claremont Road, Highgate, London, N. 6 (proposed by H. Dingle).

Eighty-four presents were announced as having been received since the last meeting, including :—

Sir Charles Vernon Boys : *The Natural Logarithm* (presented by Messrs. Wightman & Co., Ltd.).

Collection Ant. W. M. Mensing, Amsterdam (now in Chicago, Illinois). *Old Scientific Instruments (1479-1800)*. Catalogued by Max Engelmann. 2 vols. (presented by the Director of the Adler Planetarium and Astronomical Museum of Chicago).

Cordoba, Observatorio Nacional Argentino : *Resultados*, vol. xxv. "Cometa Halley" (presented by the Cordoba Observatory).

Sir Arthur Eddington : *New Pathways in Science* (presented by the author).

E. A. Milne : *Relativity, Gravitation and World-structure* (presented by the author).

The original Observation Books of the first three Radcliffe Observers—Hornsby, Robertson and Rigaud, 1774-1839 (presented by the Radcliffe Trustees).

The President announced that the following Address would be sent to H.M. the King on the occasion of his Silver Jubilee :—

To the King's Most Excellent Majesty

May it please Your Majesty

We, the President, Council and Fellows of the Royal Astronomical Society humbly beg to approach Your Majesty with the tender of our sincere and respectful congratulations on the happy completion of the twenty-fifth year of Your Majesty's auspicious reign.

Twenty-five years ago Your Majesty was graciously pleased to become the Patron of our Society. We desire to express our deep appreciation of the interest and sympathy which Your Majesty has always manifested in the intellectual well-being of Your Majesty's subjects. On behalf of all those within the wide limits of Your Majesty's Empire who are interested in the development of astronomy, we gratefully acknowledge the benefits we have enjoyed during the continuance of Your Majesty's reign, and pray that for many years Your Majesty may be spared to reign over us.

The President announced that the following had been elected by the Council as Associates of the Society :—

Professor B. P. Gerasimovič, Director of the Pulkovo Observatory ;  
Professor N. E. Nörlund, Director of the Geodetic Institute, Copenhagen ;  
Professor Svein Rosseland, Director of the Oslo Observatory ; and  
Dr. Richard Schorr, Director of the Hamburg Observatory.

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The announcement of the election of the following Associates on 1933 April 12 was accidentally omitted from the *Monthly Notices*, 93, No. 5 (1933 April) :—

Professor Bertil Lindblad, Director of the Stockholm Observatory;  
and

Dr. S. A. Mitchell, Director of the Leander McCormick Observatory  
of the University of Virginia.

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A Geophysical Discussion on "Gravity Measurements" was held at the Society's Apartments on 1935 April 26, Brigadier H. St. J. L. Winterbotham in the Chair. The Discussion was opened by Dr. E. C. Bullard.

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### THE SOLAR ROTATION AND SHIFT TOWARDS THE RED DERIVED FROM THE *H* AND *K* LINES IN PROMINENCES. (THIRD PAPER.)

J. Evershed, F.R.S.

The solar rotation derived from measures of the displacements of absorption lines at the east and west limbs of the Sun appears to increase in angular speed with the height above the photosphere. In the reversing layer I have found the stronger iron lines, and the sodium and magnesium lines tend to give rotation values 1 or 2 per cent. in excess of that given by the weaker low-level lines.

In the chromosphere Adams found that the hydrogen line *H $\alpha$*  yielded an angular velocity  $0^{\circ}.6$  in excess of the reversing layer near the equator.\* Similarly, measures of 19 spectra of the *H $\alpha$*  line obtained in 1918 at Kodaikanal gave an excess of  $1^{\circ}$  over low-level lines in the reversing layer, according to my own measures. Extending the enquiry to higher levels outside the chromosphere, the displacements of the *H* and *K* lines in prominences appear to yield much greater angular speeds.

I have summarized the results from 92 prominence spectra photographed between 1926 August and the end of 1927 in *Monthly Notices*, 88, 130, 1927, and from 289 spectra photographed during the year 1928 in *M.N.*, 89, 252, 1929. These were years of great prominence activity: according to the Kodaikanal records mean areas and mean numbers of prominences attained their highest values during those years, which was also the case with sun-spots. From 1929 to 1934 there was a progressive reduction of activity, prominences becoming less frequent and also less dense, so that many of them failed to indicate their presence on the disc of the Sun as absorption markings. Consequently it became progressively more difficult to photograph their spectra.

During the two years 1929, 1930 I photographed and measured 174 spectra, and in the four subsequent years 227 spectra. In the present paper

\* *Astrophysical Journal*, 29, 145, 1909.

I analyse and compare the results obtained during the entire period from 1926 August to 1934 December.

The autocollimating spectrograph is substantially as described in *M.N.*, 88, 127, with the addition of a  $30^\circ$  reflecting prism of 6 inches aperture lent by the Royal Society, which is used to return the light through the two 6-inch  $45^\circ$  prisms in place of a plane mirror previously used.

A carbon arc giving narrow sharply defined *H* and *K* lines is used to impress a comparison spectrum on the plates, and the small displacements of the prominence lines with reference to the arc lines are measured. The scale of the plates does not vary appreciably except when the temperature of the prism chamber changes, which it does very slowly in the course of a season. As already stated in my previous papers, the scale bears a linear relation to the wave-length over an interval of about 50 angstroms, that is to say, the most accurate measures of standard iron lines in the region fail to indicate any deviation from this relation, and the factors required at *H* and at *K* for converting the measured displacements into fractions of an angstrom are easily derived from the distance separating *H* and *K*. This interval is usually about 35 mm., or approximately 1 mm. = 1 Å. midway between *H* and *K*. A higher dispersion up to 40 mm. can be obtained without any loss of definition by setting the prisms out of minimum deviation, and this dispersion has been used for some of the brighter prominences.

As the east and west spectra are not obtained at the same time or date, each individual measure is corrected for the annual and diurnal motions of the Earth in the direction of the Sun, and the resulting shifts, expressed in km./sec., are then corrected for the revolution of the Earth and inclination of the Sun's axis. These corrections were, in the earlier series, obtained from Duner's tables, and later from the more accurate tables of Zagar published in the *Memorie della Società Astronomica Italiana*, vii, 3.

It is assumed that all prominences observed were at the Sun's limb, or  $90^\circ$  from the central meridian. As the distance from the limb cannot be exactly determined, no correction can be applied to the observed velocities, which must be slightly underestimated. On the other hand, the values of the angular rotation will be very slightly overestimated, since they decrease with the height, which also is underestimated when a prominence is not actually at the limb. These small errors in  $\xi$  therefore have opposite signs and tend to be annulled.

In any given zone of latitude on the Sun the western prominences give larger apparent rotation shifts than the eastern, owing to the general shift of the lines towards red, the rotation being given by half the sum of the shifts  $W+E$  and the general shift by half the difference  $W-E$ , reckoning the shift as a positive quantity in each case.

The prominences are found mainly between the equator and latitude  $55^\circ$ , and I have grouped them in different zones without distinction between north and south, viz. an equatorial zone with a mean latitude between  $7^\circ$  and  $9^\circ$ , and zones around  $18^\circ$ ,  $26^\circ$  and  $40^\circ$ . The limits of these zones are somewhat arbitrarily determined, according to the number of prominences found in the different regions.

TABLE I  
Group 1.—1926 August to 1928 September

| No. of Spectra | $\phi$ | $V$<br>km./sec. | $H$<br>" | $\xi$<br>° | $(W-E)/2$<br>A. |
|----------------|--------|-----------------|----------|------------|-----------------|
| 132            | 9      | 2.65            | 42       | 18.3       | +0.0131         |
| 76             | 18     | 2.34            | 31       | 16.9       | +0.0148         |
| 104            | 25     | 2.64            | 35       | 20.0       | +0.0128         |
| 69             | 40     | 1.85            | 36       | 16.5       | +0.0143         |

Group 2.—1929 February to 1934 December

|     |    |      |    |      |         |
|-----|----|------|----|------|---------|
| 124 | 8  | 2.24 | 26 | 15.6 | +0.0128 |
| 52  | 18 | 1.90 | 24 | 13.8 | +0.0103 |
| 108 | 26 | 2.08 | 31 | 15.9 | +0.0128 |
| 117 | 40 | 1.77 | 31 | 15.9 | +0.0055 |

I give in Table I the results for the entire period 1926 August to 1934 December, divided into two groups in which nearly the same number of spectra were measured. The first group represents a period of great prominence activity in which 381 spectra were measured in two years; the second group, of six years, in which I measured 401 spectra, includes the time of minimum activity.

Confining attention to the values of  $\xi$ , it is seen that there is no systematic variation with latitude. There are differences which probably have no real significance. In Group 2 the zone of  $18^\circ$  yields the minimum value, the other regions in this group giving remarkably constant values.

Assuming that in the long run the angular rotation does not change with latitude, we may take a mean value of the angular speed from a series of observations in all latitudes. Thus, in Table II such means are given, and for the purpose of comparison with the solar activity the entire series is divided into three groups representing different phases in the cycle of prominence activity.

TABLE II

|                                     | No. of Spectra | Mean Height | $\xi$<br>° | Mean Daily Number<br>(Kodaikanal) |
|-------------------------------------|----------------|-------------|------------|-----------------------------------|
| Group 1, including maximum activity | 381            | 37"         | 18.1       | 18.9                              |
| Group 2, intermediate               | 174            | 26          | 16.0       | 13.6                              |
| Group 3, including minimum activity | 227            | 30          | 15.2       | 10.9                              |

The first group includes all the measures from 1926 August to 1928 September, the second includes the years 1929 and 1930, and the third 1931 to 1934 inclusive. From the last two columns it appears that  $\xi$  decreases as the prominence activity decreases. It will be of interest to continue the work during the next period of activity, to see if the daily rotation continues at  $15^\circ$  or increases again to  $18^\circ$ .

It is possible that my method of selection of prominences with small

individual motions and narrow undistorted lines does not give values of  $\xi$  which would apply to prominences in general. This appears to be the case when comparing my results with those of E. Perepelkin. This observer, working with the 3-prism spectrograph attached to the 30-inch Pulkovo refractor has published his results for the years 1928, 1929 and 1930.\* Measuring 821 radial velocities in the lines  $Ca^+$  and 689 in the line  $H\epsilon$ , in which the random motions did not exceed 3 km./sec., he gives the angular motion  $\xi = 14^\circ \cdot 2 - 1^\circ \cdot 6 \sin^2 \phi$ , almost in agreement with the solar rotation from spots and differing entirely from my results. For random motions not exceeding 6 km./sec. he gives the result  $\xi = 13^\circ \cdot 7 - 0^\circ \cdot 9 \sin^2 \phi$ , which suggests that smaller angular speeds are found with prominences having larger proper motions, and this is confirmed by his measures of the same lines in the chromosphere, which is not subject to such large movements as the prominences. For this he gets the value  $\xi = 15^\circ \cdot 7 - 0^\circ \cdot 5 \sin^2 \phi$ , practically in agreement with my results for the prominences during the same period and showing a reduction of  $0^\circ \cdot 2$  only between the equator and latitude  $40^\circ$ .

It may be mentioned that the dispersion of the Pulkovo spectrograph is quite small, only 7 mm. between  $H$  and  $K$ , or  $1/5$ th of the dispersion of my spectrograph. It is perhaps difficult to believe that reliable mean values can be obtained with such small scale spectra in which the greatest observed shift of the lines at the Sun's equator due to rotation would not exceed  $\cdot 006$  mm., and a difference of  $1^\circ$  in  $\xi$  would amount to a difference of  $\cdot 00036$  mm. in the measures. Experience in the measurement of spectra leads me to distrust any figure that depends on differences smaller than  $1/1000$  mm., even when it is derived from a very large number of measures. Actually the difference in  $\xi$  near the equator, Evershed-Perepelkin, is  $1^\circ \cdot 5$  approximately, or on the scale of the Pulkovo plates about half a micron. It appears nevertheless from the results obtained from lines in the reversing layer with the same instrument that the prominence measures may not be affected with a considerable systematic error.

The high speed of rotation obtained by me is apparently not confirmed when the movement of prominences traversing the Sun's disc as absorption markings is measured by successive meridian transits. In the year 1910 I deduced the times of meridian transits of a prominence very favourably situated near the equator, and photographed in calcium light during three successive apparitions. From this I obtained the values  $\xi = 14^\circ \cdot 37$  and  $14^\circ \cdot 40$  from the two complete rotations. This agrees with the values obtained at Kodaikanal from a large number of  $H\alpha$  markings between lat.  $5^\circ$  and  $10^\circ$  during the years 1918 to 1929.† I also measured the apparent daily motion of the marking when near the central meridian in 1910 February and again after a rotation in March, and obtained angular motions of  $15^\circ$  and  $16^\circ$ , but this is subject to uncertainty regarding the height above the photosphere at which the absorption takes place.‡

\* *Pulkovo Observatory Circular*, No. 1, 1932.

† *Kodaikanal Observatory Bulletin*, No. 93, 1931.

‡ *Astrophysical Journal*, 33, 1, 1911.

The lack of agreement between my spectrographic measures and the angular motion derived from the times of successive meridian passages of prominences could be explained by supposing that it is the place of origin that rotates with the speed of the photosphere, whilst the prominence gases are driven forward in the direction of rotation and dissipated on emerging from the chromosphere. There is little doubt that the stable prominences are continually being renewed from below, and those of a temporary eruptive type show that the luminosity of a mass of gas cannot be maintained for more perhaps than an hour, or in some cases only a few minutes after it has been projected above the chromosphere.

A possible explanation may now be suggested of the increasing angular speed with height above the photosphere, and of the decreasing speed with decreasing solar activity. This depends upon the fact, I believe fairly well established, that the gases above the photosphere do not form a true atmosphere to the Sun, but consist of innumerable jets of luminous gas projected radially outwards from beneath the photospheric level. If the interior of the Sun is rotating faster than the photosphere, and with a uniform angular speed in all latitudes, then these jets will tend to retain the velocity of the interior regions, and the deeper the origin of a jet the more closely will its motion conform to the angular speed of the interior. The prominences coming from the deepest layers will give the greatest rotation speeds, and the most uniform angular motion. At times of intense solar activity the great prominences, streaming out at definite points in the photosphere, are probably ejected from greater depths than the smaller prominences seen at times of minimum activity. Hence their rotation speeds will be greater, as I have observed in 1927 and 1928.

The observed increase of angular speed with height above the photosphere appears therefore to indicate the opposite condition, namely, an increase of speed with depth below the photosphere.

*Shift of the H and K Lines towards Red.*—The last column on Table I gives the observed values of the shift  $(W-E)/2$  in the different zones of the two groups. Group 1 includes 92 spectra photographed in 1926–27, and in this series the values of  $(W-E)/2$  have been corrected for the error in the assumed wave-lengths of *H* and *K* referred to in *Monthly Notices*, 90, 189. The variation in the different regions probably has no significance excepting possibly in the highest latitudes around  $40^\circ$ , which give consistently low values in all the spectra of Group 2. The mean results of the whole series of 782 spectra are given in Table III, adding the shift due to the

TABLE III

| No. of Spectra | $\phi$   | $(W-E)/2$ | Mean                      | Sun-Vacuum Arc |
|----------------|----------|-----------|---------------------------|----------------|
|                |          |           | Pressure Shift for 1 atm. |                |
|                | $^\circ$ | A.        |                           | A.             |
| 256            | 9        | +0.0129   | +0.0017                   | +0.0146        |
| 128            | 18       | +0.0130   | ,,                        | +0.0147        |
| 212            | 25       | +0.0128   | ,,                        | +0.0145        |
| 186            | 40       | +0.0088   | ,,                        | +0.0105        |

pressure of one atmosphere in the arc. These are the mean values of  $H$  and  $K$  and the mean pressure shifts of the lines according to the measures of Humphreys.\* Actually  $K$  is shifted more than  $H$  in the prominences by a small amount, and it is less shifted by pressure in the arc. If the best series of measures is selected, excluding spectra having broad or ill-defined lines, the difference of shift between  $K$  and  $H$  is found to approximate very closely to the difference of pressure shift for one atmosphere according to Humphreys. In measuring the spectra I estimate the quality of the line images. Those of Class A are narrow and well defined, whilst Class B are broad and less satisfactory to measure but still considered good enough for estimates of the rotation.

If the two series of measures in all latitudes of  $K$  and of  $H$  are compared, rejecting those of Class B, the following mean results are found from 579 spectra :—

|                             | $K$       | $H$       |
|-----------------------------|-----------|-----------|
| (W-E)/2                     | +·0115 A. | +·0109 A. |
| Correction for 1 atmosphere | +·0015 A. | +·0020 A. |
| Sun—Vacuum Arc              | +·0130 A. | +·0129 A. |

The year 1928, which proved the most prolific in Class A spectra, gives the following results from 183 spectra :—

|                             | $K$       | $H$       |
|-----------------------------|-----------|-----------|
| (W-E)/2                     | +·0130 A. | +·0126 A. |
| Correction for 1 atmosphere | +·0015 A. | +·0020 A. |
| Sun—Vacuum Arc              | +·0145 A. | +·0146 A. |

This seems to confirm the relative accuracy of Humphreys' measures, if the fourth decimal in my results is to be trusted, it implies the fourth place also in millimeters.

The value +·0146 appears from Table III to be a very close approach to the truth for the lower latitudes, but by rejecting Class B measures it would be reduced to +·0140. The mean for all the A spectra including the high latitudes is as given above, +·0130. I do not give probable errors in these figures, as they would have little meaning and might give undue weight to some of the results.

As regards the high-latitude prominences, namely, those exceeding  $30^\circ$ , the high value at  $\phi = 40^\circ$  in Group 1 of Table I is very much influenced by prominences of Class B. Excluding these the mean would be reduced by ·003 A., and the mean for the whole series, including 147 spectra at mean latitude  $40^\circ$ , would then be +·0075, or excluding Class B in Group 2 also, 119 spectra give +·0070 A. This may be subject to considerable revision when more high-latitude spectra have been measured. The difference compared with low-latitude results may even turn out to be illusory, but it may be of some interest to note that outside the regions of sunspot disturbance the shift is close to the theoretical relativity shift, which is +·0081 at  $30''$  above the photosphere.

\* *Astrophysical Journal*, 26, 21, 1907.

In the reversing layer I have found no evidence of any difference of shift in spectra from low latitudes and from the Sun's poles. In the *H* and *K* region I have measured 15 iron lines in 8 polar spectra, 4 at the north pole and 4 at the south, and the result, limb - arc, corrected for 1 atmosphere is again  $+0.0146$ . This result, which includes 7 strong and 8 weak lines, agrees with the mean of the strong and weak lines in this region previously measured in low latitudes, and recorded in *Monthly Notices*, 91, 268. The prominence result  $+0.0151$  from the earlier measures is probably too large by  $0.0015$  A. Taking the lowest estimate from 579 spectra of Class A in all latitudes, it is still  $0.005$  A. in excess of the relativity shift.

The results of this research may be stated briefly as follows:—

1. The angular speed of rotation for prominences having very small random motions is found to exceed that of sunspots at the equator by at least  $1^\circ$  per day.
2. The speed appears to vary with the general activity of the Sun, being greatest at times of maximum activity.
3. The angular speed is probably constant for all latitudes.
4. An explanation of these results is suggested.
5. The general shift of the *H* and *K* lines in the prominences towards red is approximately  $+0.0146$  between the equator and latitude  $30^\circ$ , and  $+0.0075$  in higher latitudes. In the reversing layer the *Fe* lines in the same region of the spectrum give  $+0.0146$  both at the poles and in low latitudes.
6. The shift of *H* and *K* may be said to exceed the relativity shift by not less than  $0.005$  and not more than  $0.0065$  A.

## GAS MOTIONS IN PROMINENCES, WOLF-RAYET STARS AND NOVÆ.

*W. H. McCrea, M.A., Ph.D.*

### 1. *Introduction and Summary*

Gas-streaming is a familiar phenomenon in astrophysics. We encounter it in the Sun's atmosphere, notably in the motions of prominences, and it is associated on a much larger scale with Wolf-Rayet and P-Cygni stars. The intensity distribution in certain lines in the spectra of these stars can be explained only by supposing them surrounded by very extended outward-moving streams of gas, or else by entire envelopes in a state of rapid expansion. Similar manifestations of a more violent character occur in novæ. In all these cases the density of the gases can be shown to be extremely low. Nevertheless, it is remarkable that in a large number of cases, though not by any means in all, different gases like *H*, *He*, *Ca*<sup>+</sup>, *Fe*<sup>+</sup> move with almost the same velocity, even though nearly every conceivable agency that might cause the motion would be expected to act differently on these elements. It suggests itself therefore that in these cases the density is still sufficient

to prevent one gas from moving through the others with a high relative velocity. In other words, the gases may drag each other along in their motion.

The present paper gives a simple discussion of the motion of one gas through another, with application to the astrophysical phenomena described above. In particular, the conditions required to ensure small relative velocities are examined. In § 2 expressions are given for the force exerted on one gas by another gas moving through it. In § 3 the effective size of atoms for the processes considered is discussed. In § 4 a simple problem, which, however, embodies the essential features of some astrophysical cases, is formulated, and solutions of the corresponding equations of motion are written down. In § 5 some numerical cases are considered. In § 6 attention is drawn to the ways in which the treatment could be generalised.

The main conclusions are :

(a) If the density of a solar prominence is of the order of that in the lower parts of the chromosphere, then the fact that the different elements in the prominence do not separate out, but move with an almost common velocity, may be explained by the "drag" of one gas on another. The fact that  $Ca^+$  velocities are correlated to hydrogen velocities, but are slightly greater, is explained by the combined action of the "drag" and of selective radiation pressure. An appreciably smaller density would, however, permit relatively large velocities of separation of different elements. This may explain why all prominences do not show the same composition.

(b) The densities required to ensure that the gases in the expanding envelopes of Wolf-Rayet stars and novæ should move with an almost common velocity are of a quite plausible order of magnitude. But one does not know enough about the forces acting on the gases to give more definite conclusions.

## 2. *Force acting on one Gas diffusing through another Gas*

Suppose that two gases occupy the same region at uniform temperature, pressure and concentration. We require to find the force exerted by the one on the other, when they move with a given relative velocity in a given direction. This is equal and opposite to the external force which must be applied in order to maintain this relative velocity.

We adopt the following notation :—

$m_1, m_2$  = mass of molecule of gas 1, 2.

$\sigma_1, \sigma_2$  = radius of molecule 1, 2 if it can be treated as an elastic sphere.

$\nu_1, \nu_2$  = number of molecules 1, 2 per unit volume.

$D_{12}$  = coefficient of diffusion.

$u_1, u_2$  = mass velocity of gas 1, 2 in the given direction.

$U = u_1 - u_2$ , relative velocity in the given direction.

$P_1, P_2$  = external force on each molecule of type 1, 2 in the given direction required to maintain relative velocity  $U$ .

$T$  = temperature.

$k$  = gas constant.

Case (i). *U* small compared with Thermal Velocities.—In this case we can employ Chapman's \* general theory of diffusion. When the temperature, pressure and concentration are uniform his equation of diffusion (*loc. cit.* (10.02)) reduces to

$$U = \frac{D_{12}}{kT} \cdot \frac{\nu_1 + \nu_2}{\nu_1 m_1 + \nu_2 m_2} \cdot (m_2 P_1 - m_1 P_2). \quad (1)$$

Now if the motion is steady the resultant force per unit volume must vanish, so that

$$\nu_1 P_1 + \nu_2 P_2 = 0. \quad (2)$$

Also, if *F* is the reaction between the gases per unit volume,

$$F = \nu_1 P_1 = -\nu_2 P_2 \quad (3)$$

gives the force exerted by gas 1 on gas 2 in the direction of *U*. Hence

$$m_2 P_1 - m_1 P_2 = \frac{m_1 \nu_1 + m_2 \nu_2}{\nu_1 \nu_2} \cdot F.$$

Therefore (1) gives

$$F = \frac{\nu_1 \nu_2}{\nu_1 + \nu_2} \cdot \frac{kT}{D_{12}} \cdot U. \quad (4)$$

Now we may suppose that the force between the gases depends only on their temperature, densities and relative velocity, besides their intrinsic properties, and not upon their relative acceleration, nor upon what other forces may be acting as well. Hence (4) will be taken to give this force per unit volume at a point where the temperature, densities and relative velocity are given by *T*,  $\nu_1$ ,  $\nu_2$ , *U* under all circumstances, whether or not these quantities are constant in space or time, and whatever external forces may be acting.

The expression (4) is a general relation between *F* and *D*<sub>12</sub>, depending only on the assumption that *U* is sufficiently small compared with the thermal velocities. We can now introduce any suitable approximation to the coefficient of diffusion *D*<sub>12</sub>. Chapman's first approximation (*loc. cit.* (13.06)) gives

$$F = \frac{2\pi}{3} \nu_1 \nu_2 \frac{m_1 m_2}{m_1 + m_2} \cdot K_{12}'(0) \cdot U, \quad (5)$$

where *K*<sub>12</sub>'(0) depends on the law of force between the molecules. If they are regarded as elastic spheres, then Chapman's value for *K*<sub>12</sub>'(0) is

$$K_{12}'(0) = 4(\sigma_1 + \sigma_2)^2 \left\{ \frac{m_1 + m_2}{m_1 m_2} \cdot \frac{2kT}{\pi} \right\}^{1/2}, \quad (6)$$

giving

$$F = \frac{8}{3} \nu_1 \nu_2 (\sigma_1 + \sigma_2)^2 \left\{ 2\pi kT \frac{m_1 m_2}{m_1 + m_2} \right\}^{1/2} U \quad (7)$$

$$= \nu_1 \nu_2 m_1 m_2 \alpha U, \text{ say.} \quad (8)$$

\* S. Chapman, *Phil. Trans.*, **217**, A, 115-197, 1918. I am much indebted to Prof. Chapman for pointing out that his work could be utilised in this way.

*Case (ii).  $U$  large compared with Thermal Velocities.*—This case is not covered by standard investigations such as Chapman's. We consider first the case in which thermal velocities are neglected altogether. In calculating the force there is then no loss in generality if we treat, say, molecules of species 2 as at rest, and those of species 1 as all moving with uniform velocity  $U$ . We shall treat them as elastic spheres. Then the force may be simply calculated as follows:—

Consider a collision between a pair of molecules, one of each type, in which the line of centres is inclined at angle  $\theta$  to the direction of  $U$ . Then an elementary dynamical calculation shows that the gain in momentum in the  $U$ -direction experienced by molecule 2 is

$$2 \frac{m_1 m_2}{m_1 + m_2} \cdot U \cos^2 \theta. \quad (9)$$

The number of collisions per unit time in the range  $(\theta, \theta + d\theta)$  is

$$2\pi v_1 v_2 (\sigma_1 + \sigma_2)^2 U \sin \theta \cos \theta d\theta. \quad (10)$$

Hence the momentum communicated to gas 2 per unit time per unit volume, *i.e.* the force exerted on it by gas 1 per unit volume, is  $F'$ , say, obtained by multiplying (9), (10) and integrating over  $0 \leq \theta \leq \pi/2$ . This gives

$$F' = \pi v_1 v_2 (\sigma_1 + \sigma_2)^2 \cdot \frac{m_1 m_2}{m_1 + m_2} \cdot U^2 \quad (11)$$

$$= v_1 v_2 m_1 m_2 \beta U^2, \text{ say.} \quad (12)$$

This force for large values of  $U$  is to be contrasted with that given by (7) for small values of  $U$ . The different dependence on  $U$  is due to the fact that in both cases the average forward momentum communicated in a collision is proportional to  $U$ , but if  $U$  is small the number of these collisions is almost independent of  $U$ , while if it is large the number is almost proportional to  $U$ .

The crudity of the approximation (11) lies chiefly in its neglect even of the random velocities necessarily *caused* by the collisions. But the type of problem where it may be expected to give a significant first approximation is that of a relatively small cloud of heavy molecules moving with high velocity through an atmosphere of lighter molecules. For in such a case the force on the cloud is fairly accurately derived by the calculation leading to (11), since (a) the cloud is continually impinging on fresh molecules of the atmosphere, and not on those set in motion by the collisions, (b) the velocity given to a molecule 1 in a direction perpendicular to the  $U$ -direction is small compared with  $U$  when  $m_1 \gg m_2$ . It is in fact  $(2m_2/(m_1 + m_2)) U \sin \theta \cos \theta$  in the collision considered above. More accurate calculations for this case (ii) could be based on the lines of Lorentz's or Drude's treatment of the motion of electrons through a conductor in classical theory.

*Case (iii). General  $U$ .*—The problem with unrestricted values of  $U$  has been treated by R. L. Rosenberg.\* He deals with the case of generalised

\* R. L. Rosenberg, *Zs. Astrophysik*, **8**, 147–156, 1934.

elastic spheres. That is to say, he assumes that for any given relative velocity  $v$  in collision the pair of molecules behave as though they were elastic spheres of combined radius  $\sigma_1 + \sigma_2 = \sigma$ , but  $\sigma$  may be any function of  $v$ . He does suppose, however, that the molecules preserve a Maxwellian velocity distribution about their mean motion. This assumption is not made by Chapman in the work which leads to (4) for small  $U$ . Rosenberg also for simplicity takes  $m_1 = m_2$ , a trivial restriction which can easily be removed.

It is a satisfactory check that equation (7) gives the leading term of Rosenberg's result for small  $U$ , while (11) gives his leading term for large  $U$ , both for constant  $\sigma$ .

### 3. Molecular Radii

The conditions in the regions to which we later apply our results are represented by temperatures of the order  $5000^\circ$  to  $10,000^\circ$  K. (*i.e.* round about 1 volt), and pressures about  $10^{-10}$  atmosphere. Many substances are highly ionised under these conditions, so that we cannot employ the ordinary gas kinetic radii for  $\sigma_1, \sigma_2$ . Indeed, it can be only a rough approximation to use the elastic sphere model at all, and for a better approximation it would be necessary to use some other law of interaction treated by Chapman's methods. However, one of the more abundant constituents of the gas mixtures with which we have to deal is hydrogen nuclei (protons), and fortunately the collision of low-energy protons with other atoms and molecules have been extensively studied in the laboratory by Ramsauer\* and his co-workers.

These investigators find that, within the velocity range employed, the "cross-section" for proton collisions of the various atoms and molecules studied (*a*) varies considerably with the velocity, possessing a minimum in most cases between  $10\sqrt{\text{volt}}$  and  $30\sqrt{\text{volt}}$  somewhat below the gas kinetic cross-section, (*b*) increases from the minimum in these cases, with decreasing velocity, in such a way that at about  $5\sqrt{\text{volt}}$  it exceeds the gas kinetic value. The measurements do not extend below this, but presumably the cross-sections for the still smaller velocities with which we are chiefly concerned are still greater. Hence if we employ the gas kinetic cross-section in the elastic sphere model we shall be *underestimating* the interaction between the gases.

Now almost all "laboratory" values† of gas kinetic molecular radii are between  $10^{-8}$  and  $3 \times 10^{-8}$  cm. We shall here assume that the same holds good for neutral atoms in astrophysical cases. If, further, we make the same assumption for ions interacting with ions we shall again probably underestimate the effect. Hence in what follows, *when we require numerical estimates we shall use the elastic sphere model and take a round figure  $\sigma_1 + \sigma_2 = 2 \times 10^{-8}$  cm. for the radii.* Since this quantity occurs squared in

\* See in particular Ramsauer, Kollath, and Lilienthal, *Ann. d. Phys.* (5), 8, 709-736, 1931; also various papers in *Ann. d. Phys.* (5), 16, 17.

† Jeans, *Kinetic Theory of Gases*, 4th ed., 1925, chap. xiv.

formulæ (7), (11) for the interaction forces, it will lead to considerable uncertainties in our numerical results. But on the grounds set forth in this paragraph we hope to be right in our orders of magnitude, being at the same time prepared to find that we are sometimes assessing the interaction at too low a value.

#### 4. *Equations of Motion*

The following work will be recognised as akin to Rosseland's \* theory of the chromosphere. But whereas he is interested in the *structure* of the chromosphere which could be maintained by a general outward drift of positive ions, the present paper deals merely with the possible gas *velocities* in any relatively small region, in various astrophysical circumstances.

We consider again the two gases described in § 2, but we no longer suppose the motion is uniform. Instead we suppose the gases are subject to any constant forces  $m_1g_1$ ,  $m_2g_2$  (in place of  $P_1$ ,  $P_2$ ) per molecule in a constant direction. We shall suppose the mean motion is confined to this direction, and so described by  $u_1$ ,  $u_2$ , which, however, are no longer constant. We denote by  $\dot{u}_1$ ,  $\dot{u}_2$  the corresponding accelerations.

Consider the cases where the approximations (8), (12) to the interaction forces may be employed. Then the equations of mass motion are

$$\nu_1 m_1 g_1 - \nu_1 \nu_2 m_1 m_2 \gamma U^\epsilon = \nu_1 m_1 \dot{u}_1, \quad (13)$$

$$\nu_2 m_2 g_2 + \nu_1 \nu_2 m_1 m_2 \gamma U^\epsilon = \nu_2 m_2 \dot{u}_2, \quad (14)$$

where  $\epsilon = 1$  or  $2$ ,  $\gamma = \alpha$  or  $\beta$ , according as (8) or (12) is used.

Write now

$$\rho_1 = \nu_1 m_1, \quad \rho_2 = \nu_2 m_2, \quad \rho = \rho_1 + \rho_2.$$

We can then define  $V$ , which we shall call the *mean mass velocity*, by the equation

$$\rho V = \rho_1 u_1 + \rho_2 u_2. \quad (15)$$

Then from (13), (14), or from first principles, the *mean mass acceleration* of the gas mixture is given by

$$\rho \dot{V} = \rho_1 g_1 + \rho_2 g_2. \quad (16)$$

Simplifying (13), (14) and subtracting we now get the equation for the *relative velocity*  $U$ :

$$g_1 - g_2 - \rho \gamma U^\epsilon = \dot{U}. \quad (17)$$

*Case (i).*—Using (8) and integrating (17) we get

$$\rho \alpha U = g_1 - g_2 + A e^{-\rho \alpha t}, \quad (18)$$

where  $A$  is a constant of integration. Thus the *limiting relative velocity*  $U_0$  is

$$U_0 = \frac{g_1 - g_2}{\rho \alpha}. \quad (19)$$

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\* S. Rosseland, *Oslo Univ. Obs. Publ.*, No. 5, 1933.

We may also define a *time of decay*  $T_0$  of the impressed velocity, represented by the  $A$ -term in (18), by

$$T_0 = 1/\rho\alpha, \quad (20)$$

being the time required to reduce it by a factor  $1/e$ .

Case (ii).—Using (12) and integrating (17) we get

$$\sqrt{\rho\beta}U = \sqrt{g_1 - g_2} \tanh \{ \sqrt{\rho\beta(g_1 - g_2)}(t - t_0) \}, \quad (0 < \rho\beta U^2 < g_1 - g_2) \quad (21)$$

$$\sqrt{\rho\beta}U = \sqrt{g_1 - g_2} \coth \{ \sqrt{\rho\beta(g_1 - g_2)}(t - t_0) \}, \quad (0 < g_1 - g_2 < \rho\beta U^2) \quad (22)$$

valid for  $t > t_0$ , where  $t_0$  is a constant of integration. Thus with both (21), (22) the *limiting relative velocity*  $U_0'$  is

$$U_0' = \sqrt{\frac{g_1 - g_2}{\rho\beta}}, \quad (23)$$

as could have been seen from the equation of motion (17). We may define a *time of decay*  $T_0'$  as the time required for  $U$  to increase from zero to  $U_0' \tanh 1 = 0.76U_0'$ , if (21) applies, or for  $U$  to decrease from  $\infty$  to  $U_0' \coth 1 = 1.3U_0'$ , if (22) applies, by

$$T_0' = 1/\sqrt{\rho\beta(g_1 - g_2)}. \quad (24)$$

Corresponding results may be written down for  $g_1 - g_2 < 0$ , but it must be remembered that, when the sign of  $U$  changes, then the sign of the resistance terms in  $U^2$  in the equations of motion must also be changed.

The clear-cut distinction between cases (i), (ii) is, of course, an artificial one arising from the use of the two extreme forms of the interaction force. The results just given are significant approximations only if  $U$  remains sufficiently small in case (i), and remains sufficiently large in case (ii). For example, if (19) predicted a velocity  $U_0$  large compared with the thermal velocity, then we should conclude merely that we had an instance where this formula did not apply, and that consequently (23) would have to be used.

We may satisfy ourselves that this last procedure is self-consistent. For the mean thermal velocities of molecules 1, 2 are

$$W_1 = 2\sqrt{2kT/\pi m_1}, \quad W_2 = 2\sqrt{2kT/\pi m_2}, \quad (25)$$

and so a convenient mean between  $W_1$ ,  $W_2$  may be defined by

$$W = 2\sqrt{\left(\frac{2kT}{\pi} \cdot \frac{m_1 + m_2}{m_1 m_2}\right)}. \quad (26)$$

Then from (7), (11), (19), (23)

$$\frac{U_0}{W} = \frac{3}{32} \frac{m_1 m_2}{\rho k T} \cdot \frac{(g_1 - g_2)}{(\sigma_1 + \sigma_2)^2},$$

$$\frac{U_0'}{U_0} = \frac{8}{3} \sqrt{\left(\frac{2\rho k T (\sigma_1 + \sigma_2)^2}{m_1 m_2 (g_1 - g_2)}\right)}.$$

Hence

$$U_0' = \sqrt{\frac{4}{3}} U_0 W, \quad (27)$$

showing that if  $U_0 \gg W$ , then  $U_0' \ll U_0$  but  $U_0' \gg W$ .

We therefore get the simple rule: *If  $U_0$  is small compared with the thermal velocities, then  $U_0$  gives approximately the correct limiting velocity; if  $U_0'$  is large compared with the thermal velocities, then  $U_0'$  gives approximately the correct limiting relative velocity.* For in the former case  $U_0'$  is from (27) small compared with the thermal velocities, and so from its mode of derivation cannot give a significant limiting velocity. Similarly, in the latter case  $U_0$  is from (27) large compared with the thermal velocities and so it cannot give a significant limiting velocity. Thus there is no ambiguity such as would occur if  $U_0, U_0'$  both gave possible limiting velocities for the same gas mixture. Nor is there any inconsistency such as would occur if it were possible to have  $U_0$  large compared with the thermal velocities, and  $U_0'$  small compared with them, so that neither would give a significant limiting velocity.

### 5. Applications.

The difficulty in obtaining instructive numerical values is that so many parameters enter that we have to know a fair amount about them all in order that the uncertainty in the final result may not be overwhelming. So it is best to deal with a few particular cases, rather than attempt to classify general conclusions.

*Solar Prominences.*—Suppose tentatively that a solar prominence consists of an excess of hydrogen, at  $6000^\circ$  K., say, for which the force of gravitation is almost balanced by some other force. We first examine the conditions under which it can carry up other gases with it, these being subject to no external force other than gravity. Consider the case of neutral helium.

Let gas 1 be  $H$ , and gas 2 be  $He$ . Then our assumptions amount to

$$\begin{aligned} m_1 &= H, \text{ the mass of the } H \text{ atom,} & m_2 &= 4H, \\ \nu_1 &\gg \nu_2, \text{ so that } \rho = \nu_1 H \text{ approximately,} \\ \sigma_1 + \sigma_2 &= 2 \times 10^{-8} \text{ cm. from § 3,} \\ g_1 &= 0 \text{ approximately,} \\ g_2 &= -g, \text{ the value of gravity at the Sun's surface.} \end{aligned}$$

Then, inserting numerical values in our various formulæ, we find for this case

$$U_0 = \frac{6.5 \times 10^{13}}{\nu_1}, \quad (28) \qquad T_0 = \frac{2.4 \times 10^9}{\nu_1}, \quad (29)$$

$$U_0' = \frac{1.0 \times 10^{10}}{\sqrt{\nu_1}}, \quad (30) \qquad T_0' = \frac{3.8 \times 10^5}{\sqrt{\nu_1}}. \quad (31)$$

Thus, in order, say, to have  $U_0 < 5$  km./sec., assuming (28) holds good up to such a velocity, the mean thermal velocity of  $H$  under the conditions being about 10 km./sec., we should have  $\nu_1 > 1.3 \times 10^8$  particles/cm.<sup>3</sup>.

Also, in order, say, to have  $U_0' > 20$  km./sec., assuming (30) holds good down to such a velocity, we should have  $\nu_1 < 2.7 \times 10^7$  particles/cm.<sup>3</sup>.

Further, if  $\nu_1 = 1.3 \times 10^8$  in (29), then  $T \sim 18$  sec., and if  $\nu_1 = 2.7 \times 10^7$  in (31), then  $T_0' \sim 73$  sec. This serves to show that the limiting velocities will be approximated to in very short times, and consequently after the gases have moved quite short distances.

Now the density in a prominence must be something like that in the chromosphere.\* Rosseland (*loc. cit.*, p. 24) obtains an electron density  $2 \times 10^8$  per cm.<sup>3</sup> at the top of the chromosphere, while I have obtained † one of  $2 \times 10^{10}$  per cm.<sup>3</sup> at the base. Menzel's estimate (*loc. cit.*, p. 294) is still higher. Now there must be present an almost equivalent number of positive ions. These may be taken to be largely *protons*, and so contributing to  $\nu_1$  in the case considered. It is more difficult to estimate the contribution of neutral *H* atoms. But even if the latter is small we can still draw the following conclusions:—

If the density in a prominence is of the same order as in the lower parts of the chromosphere, then a gas like *He* must share to within a small fraction the general velocity of the hydrogen. On the other hand, we see that the conditions may be somewhat critical. For the above values of  $U_0$ ,  $U_0'$  show that if the density drops from about  $10^8$  to about  $10^7$  atoms/cm.<sup>3</sup>, then *He* would certainly lag behind the *H*. This gives indeed a clue to the apparent difference of composition of different prominences. For example, Menzel (*loc. cit.*, p. 295) records a striking case of a large eruptive prominence in which all the elements exhibited exactly the same details of structure, whereas in certain cases there does appear to be a tendency for different elements to separate out.‡

We are, of course, as far as ever from knowing what the general propelling force in prominences may be. But at any rate our assumption  $g_1 \sim 0$ , making  $V \sim 0$  in (16), is consistent with the observed fact that, apart from occasional abrupt changes, the motion of eruptive prominences is uniform.§

Now suppose some gas in a prominence is subject to selective radiation pressure. Consider the case of  $Ca^+$ , now taken to be gas 2, and suppose that the net force per atom away from the Sun is such that

$$g_2 = kg \quad (k \text{ constant}).$$

We still make the same assumptions about *H* (gas 1). Then we find on inserting numerical values,

$$U_0 = 5.9 \times 10^{14} k / \nu_1, \quad (32)$$

$$U_0' = 2.9 \times 10^{10} \sqrt{(k/\nu_1)}, \quad (33)$$

where  $U_0$ ,  $U_0'$  are now the velocities of  $Ca^+$  relative to *H*, away from the Sun.

The force due to selective radiation pressure in such cases is generally

\* D. Menzel, *Lick Obs. Publ.*, 17, pt. 1, 297, 1931; S. Chandrasekhar, *M.N.*, 94, 14-35, § 17, 1934.

† W. H. McCrea, *M.N.*, 89, 483-497, 1929.

‡ E. Pettit, *P.A.S.P.*, 45, 15, 1933.

§ E. Pettit, *Ap. J.*, 76, 9, 1932.

taken to be such that  $k$  is of the order of unity.\* So we see that if  $\nu_1$  is, say,  $10^{10}$  atoms/cm.<sup>3</sup>, then the relative velocity is given by (32) and is  $U_0 \sim 0.6$  km./sec., while if  $\nu_1$  is, say,  $10^8$  atoms/cm.<sup>3</sup> the relative velocity must be taken from (33) and is  $U_0' \sim 30$  km./sec.

Now there is good indication that the state of affairs with respect to  $Ca^+$  in prominences is not very remote from the sort of thing just imagined. For  $H$  and  $Ca^+$  do move together,† but while Perepelkin finds that their velocities are strongly correlated, he finds also that of  $Ca^+$  is in general somewhat greater than that of  $H$ . This suggests that a selective force acts on  $Ca^+$ , but that the resistance of the  $H$  is sufficient to maintain a limiting relative velocity and prevent separation.

*Wolf-Rayet Stars.*—Beals's‡ theory of Wolf-Rayet emission suggests that a star of this type is surrounded by an envelope or shell of gas expanding with a velocity of the order of 1000 km./sec. It also appears that different elements move with similar velocities; see, for example, the case of  $H$  and  $He^+$  in Beals's Table VI (*Publ. D.A.O., loc. cit.*).

Let us suppose for the moment that the conditions are qualitatively like those discussed above for  $H$ ,  $He$  in the case of the Sun. Suppose further that the mass of a typical Wolf-Rayet star is the same as the solar mass. Then if (23) applies, we have from (30)

$$\nu_1 = \frac{10^{20} \left( \frac{R_\odot}{R} \right)^2}{U_0'^2}, \quad (34)$$

where  $R$  is the radius of the expanding shell, and  $R_\odot$  the solar radius. Thus, if the relative velocity  $U_0' \sim 100$  km./sec., then for  $R = 10R_\odot$ ,  $\nu_1 = 10^4$  particles/cm.<sup>3</sup>, and for  $R = 100R_\odot$ ,  $\nu_1 = 10^2$  particles/cm.<sup>3</sup>. The actual densities cannot well be as small as this. So these figures suggest that the agency under consideration may be operative in preventing a large spread of velocities in the shell. They show that if hydrogen were driven off with a velocity  $\sim 1000$  km./sec. in the quantities indicated by the values of  $\nu_1$ , then any helium originally mixed with it would be carried out in its train with a comparable velocity, and would not fall back under the influence of the star's gravitation.

If the gases were merely ejected from the star with velocities greater than the velocity of escape, and subsequently moved away under no external force other than gravity, then since  $g_1 - g_2 = 0$  the relative velocities of the gases would tend to zero. But selective radiation pressure due to fluorescence in the manner contemplated by Zanstra§ and others is probably always of importance in these cases of hot stars. This makes the motion of the hydrogen, which doubtless can always be assumed the most abundant element present, much less difficult to understand here than, say, in a solar prominence.

\* S. Chandrasekhar, *loc. cit.*, § 3; S. R. Pike, *M.N.*, **88**, 635-640, 1928.

† E. Pettit, *Ap. J.*, *loc. cit.*; E. J. Perepelkin, *Zs. Astrophys.*, **1**, 326-330, 1930.

‡ C. S. Beals, *M.N.*, **90**, 202-212, 1930; *Publ. D.A.O. (Victoria)*, **4**, No. 17, 1931; see also W. H. McCrea, *Zs. Physik.*, **57**, 367-379, 1929.

§ H. Zanstra, *M.N.*, **95**, 84-101, 1935.

*Novæ.*—It is well known that the variations and complexities in novæ spectra can only be explained by postulating the ejection of matter in one or more expanding shells. The whole phenomenon is very complicated,\* and it must suffice to consider one example. Spencer Jones found that at one stage in the evolution of Nova Pictoris (1925), just after maximum, there were expanding shells with velocities about 100 and 300 km./sec., approximately common to all elements, but with *H* somewhat slower than the others. He estimated in addition that at maximum the radius of the nova was 384 solar radii.

If we apply (34), again assuming the solar value for the mass, and take, say,  $U_0' = 20$  km./sec., setting  $R = 400R_\odot$ , then  $v_1 \sim 156$ . Or, if we apply (33) with  $U_0' = 20$  km./sec.,  $k = 1$ ,  $R = 400R_\odot$ , then  $v_1 \sim 1330$ . Thus, under the assumed conditions a density of hydrogen of the order of 200 atoms/cm.<sup>3</sup> would be sufficient to prevent *He* from lagging behind by as much as 20 km./sec., while a density of the order of 2000 atoms/cm.<sup>3</sup> would prevent  $Ca^+$  from having a velocity 20 km./sec. in excess of *H*. These densities are of the order of those in diffuse nebulae, so that the actual densities associated with the nova can scarcely be as low, and therefore the force of interaction between the gases must really be greater than we have allowed. The values assumed for the external forces are, however, little more than guesswork. But the results again show at least the possible importance in this field of the phenomenon considered.

## 6. Generalisations

The first type of elaboration needed in any more detailed application of the theory is a better treatment of the molecular collisions leading to a formulation of the interaction forces  $F, F'$ . Either laboratory values of the "effective" ( $\sigma_1 + \sigma_2$ ) for the particular atoms or ions concerned, or a theoretical treatment of the collisions under an appropriate law of force, should be used.

Secondly, more exact equations of motion allowing density and pressure gradients may be needed. Also in problems of expanding shells of gas the fact that external forces must depend on the distance from the star must be taken into account in writing down these equations.

Finally, we have discussed only what would happen once the various gases are mixed together. We have not considered any mechanism of projection to see if the gases are likely to be suitably mixed to begin with.

I am greatly indebted to Professor Stratton for his guidance in regard to observational data. He must not, however, be held responsible for any interpretations I have ventured to put upon them.

*Imperial College of Science :*

1935 April.

\* F. J. M. Stratton, *Ann. Solar Phys. Obs. Cambridge*, 4, pt. 1, 1920; W. H. Wright, *Publ. Lick Obs.*, 14, pt. 2, 1926; H. Spencer Jones, *Ann. Cape Obs.*, 10, pt. 9, 1931.

## THE VARIABLE SPECTRUM OF $\gamma$ CASSIOPEIÆ. (THIRD PAPER.)

*William J. S. Lockyer, M.A., Ph.D., Major (late R.A.F.).*

In my first paper on the spectrum of  $\gamma$  Cassiopeiæ (H.D. 5394, 1900 R.A. 0<sup>h</sup> 50<sup>m</sup>.7, Dec. +60° 11', Ptg. Mag. 2.01) communicated to the Society on 1933 March \* I showed that this spectrum was variable, as was indicated by the changes in relative intensity of the bright components of the hydrogen lines; it was pointed out (p. 363) that distinct changes of intensity had also been detected in the behaviour of other lines in the spectrum, although no specific data were given.

The observations there discussed included all those made up to the end of 1933 February, and it was not intended to publish anything further about this star for some years to come. The reason for this was that, as a periodic variation of about four years' duration seemed to be in operation, and as a minimum of the curve appeared to have been reached about the beginning of the year 1933, the programme intended was to follow the spectral changes throughout another complete cycle which was expected to terminate about the beginning of the year 1937.

Now the new observations have shown that not only was the forecasted minimum for 1933 not reached, but the spectrum about this period underwent an extraordinary change. This change is, so far as I can ascertain, perfectly unique in the history of this star, and a similar change has never occurred in the spectra of any of the other Be-type stars, the spectra of many of which I have studied.

In the present discussion the spectra were all taken with the same instrument as before, namely, the prismatic camera of 9 inches aperture, 120 inches focal length, with a prism of 45° angle giving a dispersion of 46.0 mm. between  $H\beta$  and  $H\delta$ .

The number of new spectra taken total 59 and cover the period from 1933 March 18 to 1935 March 11, two or more photographs being secured generally every month.

This paper, like the previous one, is limited strictly to the behaviour of the hydrogen lines, although changes also occurred in other lines of the spectra.

The methods of determining the differences between the intensities of the bright components of the hydrogen lines were the same as in the previous paper, namely, by eye estimate and wedge measurement. Owing to the new change in the spectrum, the notation previously given (p. 366) for both the eye estimates and the wedge measures had to be extended at both ends to include new values outside the previous range.

\* *M.N.*, 93, 362, 1933.

This extension of the range for the eye estimates is shown in fig. 1, where the prolongation of the scale on the left is indicated on the right with the appropriate notation. Previously the limit of the positive values was 5, but the new spectra have necessitated the addition of the values +8, +11 and +14.

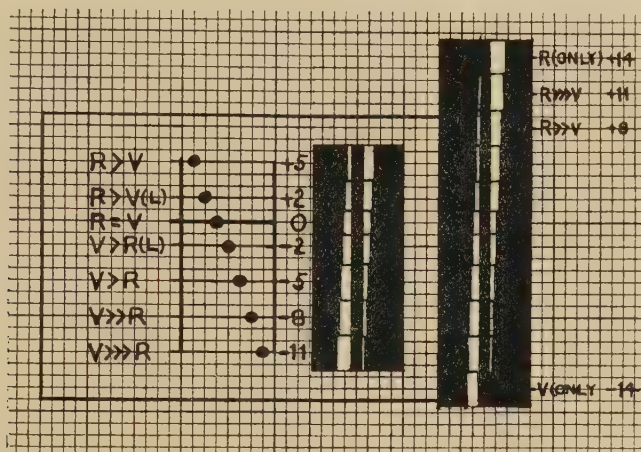


FIG. 1.

As regards the negative values, the bright red component (R) in some of the photographs disappears altogether, so that the value given for the violet component (V) being present alone has been adopted as -14.

In the case of the estimates made with the eye, using a magnifying glass ( $\times 2$ ), the differences of intensities were found for each of the hydrogen lines  $H\beta$ ,  $H\gamma$ ,  $H\delta$ ,  $H\epsilon$  and  $H\zeta$ . The bright components of  $H\beta$  are so close together, and the images so strong, that the differences could only be detected when they were very great: the values for these are omitted therefore from the subsequent table. The estimates for  $H\zeta$  are a little less certain than those for the intermediate lines.

In Table I, which follows, the letters "M" and "P" refer to the quality of the estimates made for each pair of the bright components, depending chiefly on the definition of the lines, these letters representing "moderate" and "poor" respectively. The absence of a letter means that the estimate is "good."

As a check on the eye estimates, measures were made quite independently with the wedge micrometer by Mr. D. L. Edwards, but these were restricted to the  $H\gamma$  line. Owing to the higher magnification used, measures were not possible with all the available negatives but only with the best. The values derived are given in the last column but one in Table I, followed by the quality of the measures, "P" representing "poor" measures: in all other cases the measures were "good."

Table I shows the values secured by both methods. The first three columns give the Sidmouth negative number, the year and date respectively.

In the other columns E, Q and M represent the eye estimates, quality of definition and wedge measures respectively.

Attention should be drawn to the dates 1933 December 31 and 1934 January 19 and 29 in the case of the wedge measures. Against these the symbol (R?) is placed in the last column but one. This indicates that the red component was just visible but too feeble to measure.

From 1934 March 27 to November 2 the red component had completely vanished, leaving the violet component alone: this condition is indicated by (Ro). It will be shown later on that for the last three months in the table for 1934 (Vo) should really be substituted for (Ro).

TABLE I

| Neg.<br>No. | Year | Date    | Eye Estimates |   |           |   |             |   |          |   | Wedge<br>Measures |   |
|-------------|------|---------|---------------|---|-----------|---|-------------|---|----------|---|-------------------|---|
|             |      |         | $H\gamma$     |   | $H\delta$ |   | $H\epsilon$ |   | $H\zeta$ |   | $H\gamma$         |   |
|             |      |         | E             | Q | E         | Q | E           | Q | E        | Q | M                 | Q |
| 1926        | 1933 | Mar. 18 | - 8           |   | - 11      |   | - 11        |   | - 11     |   | - 0.6             |   |
| 1936        | "    | " 25    | - 8           |   | - 11      |   | - 11        |   | - 11     |   | - 0.3?            | P |
| 1942        | "    | Apr. 5  | - 8           |   | - 11      |   | - 11        |   | - 11     |   | - 0.8             | P |
| 1952        | "    | " 15    | - 8           |   | - 11      |   | - 11        |   | - 11     |   | - 0.5             |   |
| 1959        | "    | May 3   | - 8           |   | - 11      |   | - 11        |   | - 11     |   |                   |   |
| 1960        | "    | " 4     | - 8           |   | - 11      |   | - 11        |   | - 11     |   |                   |   |
| 1965        | "    | " 22    | - 8           |   | - 11      |   | - 11        |   | - 11     |   |                   |   |
| 1966        | "    | " 23    | - 8           |   | - 11      |   | - 11        |   | - 11     |   | - 0.5?            | P |
| 1972        | "    | June 5  | - 8           |   | - 11      | M | - 11        |   | - 11     | M |                   |   |
| 1976        | "    | " 20    | - 8           | M | - 8       | P |             |   |          |   |                   |   |
| 1977        | "    | " 20    | - 8           | M | - 11      | M | - 11        | P |          |   |                   |   |
| 1983        | "    | July 1  | - 8           |   | - 11      |   | - 11        |   | - 11     |   | - 0.9             |   |
| 1986        | "    | " 5     | - 8           |   | - 11      |   | - 11        |   | - 11     |   | - 1.0             |   |
| 1989        | "    | " 17    | - 8           |   | - 11      |   | - 11        |   | - 11     |   | - 0.8             |   |
| 1998        | "    | Aug. 4  | - 8           |   | - 11      |   | - 11        |   | - 11     |   | - 1.1             |   |
| 1999        | "    | " 4     | - 8           |   | - 11      |   | - 11        |   | - 11     |   |                   |   |
| 2021        | "    | " 27    | - 8           |   | - 11      |   | - 11        |   | - 11     |   | - 0.7?            | P |
| 2026        | "    | Sept. 2 | - 8           |   | - 11      |   | - 11        |   | - 11     |   |                   |   |
| 2028        | "    | " 5     | - 11          |   | - 11      |   | - 11        |   | - 11     |   |                   |   |
| 2046        | "    | " 23    | - 11          |   | - 11      |   | - 11        |   | - 11     |   | - 1.5?            | P |
| 2062        | "    | Oct. 3  | - 11          |   | - 11      |   | - 11        |   | - 11     |   | - 1.7?            | P |
| 2066        | "    | " 8     | - 11          |   | - 11      |   | - 11        |   | - 11     |   | - 1.0             |   |
| 2101        | "    | Nov. 25 | - 11          |   | - 11      |   | - 11        |   | - 11     |   | - 1.5             |   |
| 2117        | "    | Dec. 14 | - 11          |   | - 11      |   | - 14        |   | - 14     |   | - 1.8             |   |
| 2122        | "    | " 31    | - 11          |   | - 11      |   | - 14        |   | - 14     |   |                   |   |
| 2123        | "    | " 31    | - 11          |   | - 14      |   | - 14        |   | - 14     |   | R?                |   |
| 2124        | "    | " 31    | - 11          |   | - 14      |   | - 14        |   | - 14     |   |                   |   |
| 2134        | 1934 | Jan. 19 | - 11          |   | - 11      |   | - 14        |   | - 14     |   | R?                |   |
| 2141        | "    | " 29    | - 11          |   | - 11      |   | - 14        |   | - 14     |   | R?                |   |
| 2149        | "    | Feb. 6  | - 11          |   | - 11      |   | - 14        |   | - 14     |   | R?                |   |
| 2172        | "    | " 25    | - 11          |   | - 11      |   | - 14        |   | - 14     |   | R?                |   |

| Neg.<br>No. | Year | Date    | Eye Estimates |   |           |   |             |   |          |   | Wedge<br>Measures |   |
|-------------|------|---------|---------------|---|-----------|---|-------------|---|----------|---|-------------------|---|
|             |      |         | $H\gamma$     |   | $H\delta$ |   | $H\epsilon$ |   | $H\zeta$ |   | $H\gamma$         |   |
|             |      |         | E             | Q | E         | Q | E           | Q | E        | Q | M                 | Q |
| 2195        | 1934 | Mar. 27 | -14           | M | -14       |   | -14         |   | -14      |   | Ro                |   |
| 2210        | "    | Apr. 9  | -14           | M | -14       |   | -14         |   | -14      |   | Ro                |   |
| 2211        | "    | " 9     | -14           | M | -14       |   | -14         |   | -14      |   | Ro                |   |
| 2215        | "    | " 20    | -14           | M | -14       |   | -14         |   | -14      | M | Ro                |   |
| 2219        | "    | " 22    | -14           | M | -14       |   | -14         |   | -14      |   | Ro                |   |
| 2223        | "    | " 30    | -14           | M | -14       |   | -14         |   | -14      | P | Ro                |   |
| 2227        | "    | May 14  | -14           | M | -14       |   | -14         |   | -14      |   | Ro                |   |
| 2231        | "    | " 23    | -14           |   | -14       |   | -14         |   | -14      | M | Ro                |   |
| 2236        | "    | " 28    | -14           |   | -14       |   | -14         |   | -14      |   | Ro                |   |
| 2240        | "    | " 29    | -14           |   | -14       |   | -14         |   | -14      |   | Ro                |   |
| 2247        | "    | July 1  | -14           |   | -14       |   | -14         |   | -14      | P | Ro                |   |
| 2249        | "    | " 4     | -14           |   | -14       |   | -14         |   | -11      | M | Ro                |   |
| 2256        | "    | " 23    | -14           |   | -14       |   | -14         |   | -11      | P | Ro                |   |
| 2262        | "    | Aug. 11 | -14           |   | -14       |   | -14         |   | -11      | P | Ro                |   |
| 2266        | "    | " 15    | -14           |   | -14       |   | -14         |   | -11      |   | Ro                |   |
| 2271        | "    | " 29    | -14           |   | -14       |   | -14         |   | -11      |   | Ro                |   |
| 2277        | "    | Sept. 5 | -14           |   | -14       |   | -14         |   | -14      | M | Ro                |   |
| 2283        | "    | " 26    | -14           |   | -14       |   | -14         |   | -14      |   | Ro                |   |
| 2291        | "    | Oct. 5  | -14           |   | -14       |   | -14         |   | -14      |   | Ro                |   |
| 2306        | "    | Nov. 2  | -14           |   | -14       |   | -14         |   | -14      |   | Ro                |   |
| 2315        | "    | Dec. 20 | +11           |   | +11       | M | +11         | M | +11      |   | ...               |   |
| 2322        | "    | " 26    | +11           |   | +11       |   | +11         |   | +11      |   | +1.8              |   |
| 2335        | 1935 | Jan. 7  | +11           |   | +11       |   | +11         |   | +11      |   | +2.1              |   |
| 2345        | "    | " 18    | +11           |   | +11       |   | +11         |   | +11      |   | +1.5              |   |
| 2366        | "    | Feb. 10 | +11           |   | +11       |   | +11         |   | +11      |   | +1.6              |   |
| 2368        | "    | " 23    | +11           |   | +11       |   | +11         |   | +11      |   | +2.1              | P |
| 2375        | "    | " 28    | +11           |   | +11       |   | +11         |   | +11      |   | +2.2              |   |
| 2387        | "    | Mar. 11 | +11           |   | +11       |   | +11         |   | +11      |   | +2.0              |   |

From Table I the diagram (fig. 2) has been formed. The values plotted are the monthly mean values for the eye estimates of  $H\gamma$  and  $H\delta$  and the wedge measures of  $H\gamma$ . The eye-estimate values for  $H\epsilon$  and  $H\zeta$  have not been included, as the curves formed from them corroborated those for  $H\gamma$  and  $H\delta$ .

Some of the data previously published,\* commencing in 1928 August 26 and ending in 1933 February 14, have also been plotted, so as to give an indication of the trend of the curves prior to the new series of observations. The filled-in and open circles represent good and moderate observations respectively. The time scale is given at the top, the thick vertical lines indicating the month of January of each year. Similar lines for the other

\* M.N., 93, 367, 1933.

months have been omitted. The scales for intensity differences are shown on the left.

Smooth curves have been drawn through each set of measures to satisfy the observations as nearly as possible. These, it will be seen, are alike in

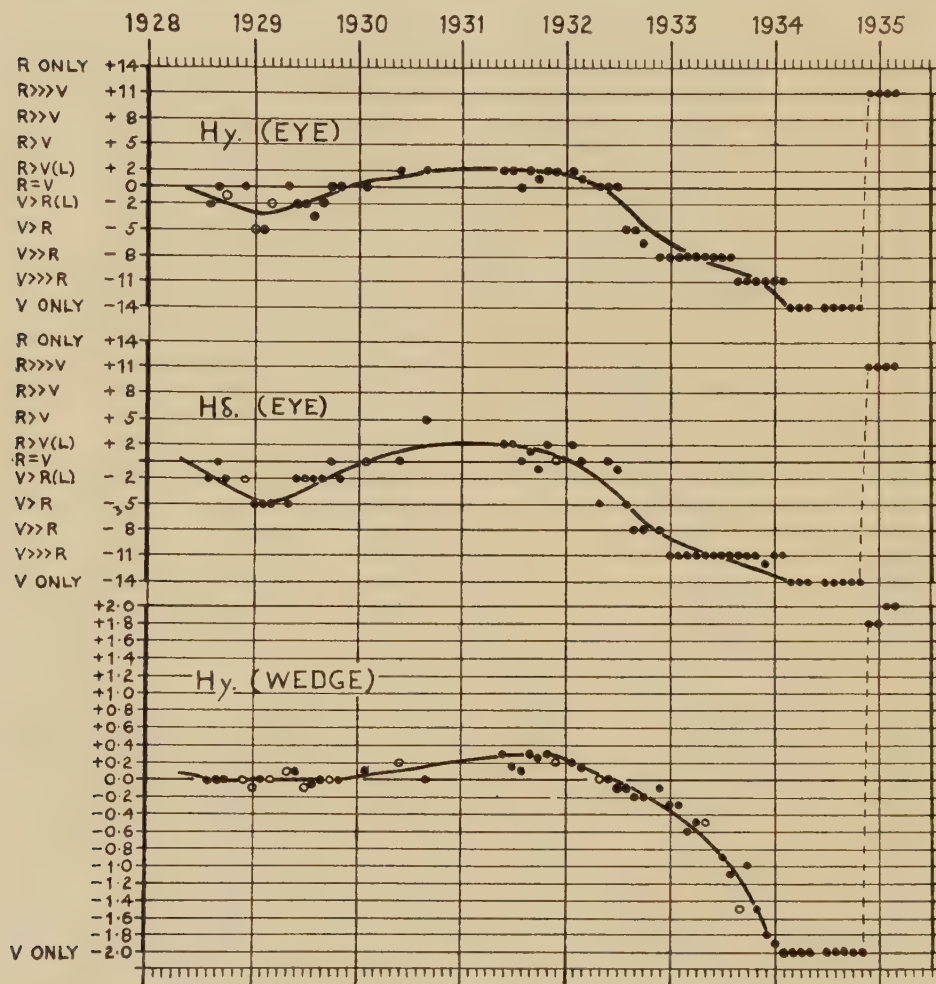


FIG. 2.

their main features, indicating a shallow minimum at about the beginning of the year 1929, a maximum about the middle of 1931 and another minimum, in this case very deep, about the year 1934.

In my previous paper the trends of the curves were such that a minimum was expected in about the year 1933, but subsequent observations, as here shown, have negated this.

Attention should be drawn to the bottoms of all the three scales which are marked "V only." When observations are plotted to these values, as indicated in the year 1934, they mean that the red components of the

hydrogen lines have *completely vanished*, leaving a single bright line alone, and of very strong intensity.

It was thought that as the red component had vanished in 1934 March later observations would show the gradual reappearance of this component, growing stronger as the violet component diminished in intensity. This has been the usual feature in all the Be-type stars which I have studied, to give only two examples,  $\phi$  Persei \* and H.D. 20336 † (Camelopardalis), curves for which have been previously published.

Such, however, was not the case. Instead of the *red* component coming into view again as a *weak* line it appeared *very strong* indeed, equally as strong as the single bright line had appeared during the previous months. The violet component, on the other hand, showed only as a *very weak* line. This extraordinary and unique condition of affairs was totally unexpected.

Instead therefore of the curves (fig. 2) rising again to a maximum they all became discontinuous, and a new curve commenced at the other extreme end of the scales. This behaviour of the curves reminded one at once of the well-known curves for the changes of latitude of sunspots from year to year, about the time of minimum spot activity, only here there is no overlapping of the high- and low-latitude curves. For the four months from 1934 December up to the present time (March) the red component is very much brighter ( $R \gg V$ ) than the violet component.

The question arises in what manner did this apparently sudden change-over from a strong violet component with no red component to a strong red component with a weak violet component take place.

Did the strong violet component disappear suddenly and the strong red component appear suddenly at their respective wave-lengths, or was there some intermediate stage?

It should be remembered that each of the hydrogen lines, namely,  $H\gamma$ ,  $H\delta$  and  $H\epsilon$ , during the long period from 1934 March 27 to November 2 exhibited only one strong bright line. Was it possible that during those months this bright line, which was the violet component in 1934 February, maintained its intensity but gradually moved towards the red end of the spectrum, and just at the end of this period, namely, 1934 December 20, reached the position where the red component should appear?

Of course such a change could easily have been verified if the spectra had been taken with a slit spectroscope having a comparison spectrum, but as they were all secured with a prismatic camera no such test was possible.

Another method, however, was available. It was first of all known that just before the red component vanished the next succeeding photographs showed only one component and this was undoubtedly the violet component. This was done by placing the negatives film to film and adjusting them so that *all the other lines in the spectrum coincided*: the coincidence of this single line with the violet component was obvious (see Plate 7).

It was concluded therefore that, if all the negatives were in turn placed against each other with respect to all lines *other than hydrogen*, the change of

\* M.N., 86, 487, 1926.

† M.N., 91, 220, 1930.

position of the single bright hydrogen line could be observed. An attempt was therefore made to investigate this point.

A good negative, taken in 1934 December 26, was selected to represent the latest condition of the spectrum, *i.e.* one showing strong red components and weak violet components of the hydrogen lines. This negative was placed film to film with each of twelve selected negatives (out of twenty available) taken between 1934 March and November, in which only one bright hydrogen line was visible, and all the lines other than hydrogen made coincident with each other.

The negatives were compared in order of time, and it was found that, at the beginning of the series, the single bright line coincided with the violet component, and at the end of the series with the red component (see Plate 7). At the intermediate stages the single bright line did not coincide in position with either of the components, but took up positions between them. Thus a distinct *creep* of the single bright line from its initial position opposite the violet component to its final position opposite the red component was observed.

In the light of these facts, it will be seen that in Table I, while the single bright lines of  $H\gamma$ ,  $H\delta$  and  $H\epsilon$  in 1934 March and April are correctly marked as  $(-14)$  representing the violet component being visible alone, the bright lines in the months 1934 September to November should have been marked  $(+14)$  as they are now seen to represent the red component alone.

Attention should be drawn to the behaviour of the line  $H\zeta$  between the dates 1934 July 4 and August 29. It will be seen from Table I that during this interval the two components are recorded as being double.

It should be remembered that at about this wave-length the sensitiveness of the plate begins to fall off and the bright lines also become less intense towards the ultraviolet, so that eye estimates have a little less weight, as I have already pointed out.

There nevertheless seems to be something peculiar about the  $H\zeta$  line at this time, because not only does it seem to be brighter for its position with regard to the other hydrogen lines than would be expected but it also appears broader.

A very detailed examination of all the negatives taken about this time suggests, however, that the apparent anomaly of this line may be due to a photographic rather than a real origin.

It is interesting now to make a comparison between the behaviour of the hydrogen lines in the spectrum of  $\gamma$  Cassiopeiæ and those in normal Be-type stars throughout a full cycle: this can best be accomplished by the accompanying diagram (fig. 3).

The broad absorption band of hydrogen is shown in each case by a wide black band, and the narrow absorption line by a black line down the centre. The bright lines are represented as situated on each side of this narrow absorption line, and their relative intensities are indicated by their widths. The red and violet sides of the lines are on the right- and left-hand sides of the hydrogen lines respectively.

The hydrogen line on the left (marked A) illustrates the general behaviour

of each of these lines in  $\gamma$  Cassiopeiae, the scale of years, with each month indicated, being shown vertically on the left and the eye estimates of the relative intensity of the bright lines towards the right.

The hydrogen line on the right (marked B) represents the behaviour of the hydrogen lines as exhibited in the spectra of what may be termed normal Be-type stars and which it was thought would have been observed in the spectrum of  $\gamma$  Cassiopeiae.

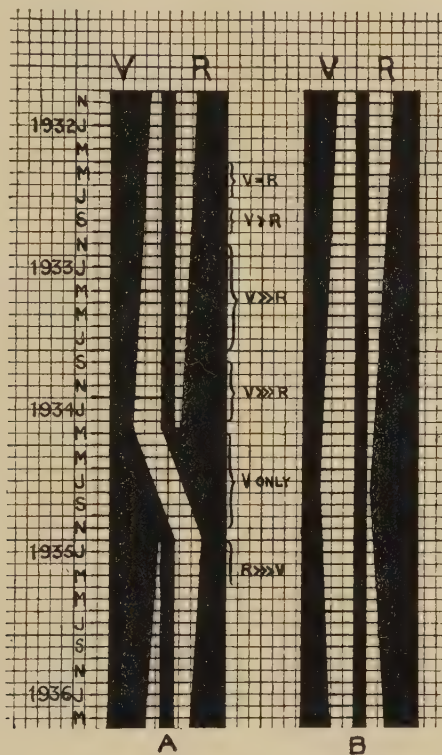


FIG. 3.

In the case of the normal Be-type spectrum (marked B) it will be seen that at the top the bright red component of the hydrogen line is showing stronger than the violet. As one proceeds downwards (in time) the violet component becomes stronger than the red until a maximum of the violet is reached (or a minimum of the red). After that the red component begins to increase its intensity again and the violet to decrease.

Now in the case of  $\gamma$  Cassiopeiae, as shown on the left-hand side (marked A), the red component at the top (1932 January) is stronger than the violet component. This red component gradually reduced its intensity while the violet increased, until in 1934 March the red component disappeared altogether, leaving the violet component very strong and alone and in the position of the violet component. The diagram illustrates the gradual "creep" of this violet component towards the position of the red during the year 1934. It shows also the appearance of the faint violet component

in 1934 December and the probable future relative intensities that they will display.

The above comparison brings out the eccentric nature of the  $\gamma$  Cassiopeiæ hydrogen lines, and it will be most interesting to watch the future changes to see whether the condition of 1934 March will repeat itself or not as the case may be.

So far as can be seen at present the theories suggested for the origins and behaviour of the bright lines in Be-type stars do not seem capable of explaining this peculiar behaviour of these lines in  $\gamma$  Cassiopeiæ.

Otto Struve,\* in 1931, published an hypothesis to account for the bright lines in Be-type stars. The suggestion he offered was that rapidly rotating single stars of this class are unstable, and form lens-shaped bodies which eject matter at the equator. Thus a nebulous ring is formed which revolves around the star and gives rise to the emission lines. The inclination of the star's axis is held responsible for the observed range in the width of the broad emission lines. To explain the changes in the intensities of the bright components he suggests that the emitting body may not be a very stable one. Further, if the orbits of the individual atoms in the ring are elliptical instead of circular, this would tend to make the intensities of the two components unequal, and a rotation of the line of apsides of the elliptical orbits would cause a periodic variation in the intensity of the two components.

Dean McLaughlin,† while adopting Struve's suggestion that the atmosphere of a Be star is of very great extent compared with the star itself, and that the considerable width of the emission lines is due to rapid rotation of the stellar atmosphere, explains in a different manner the changes in the relative intensity of the emission components.

He regards the atmosphere of the star as of a pulsating nature, which in combination with the rapid rotation would result in differential Doppler effects between the emitting and absorbing atoms. With an expanding atmosphere the outer layer of absorbing atoms would all have radial velocities of approach and would tend to absorb emission lines from atoms with similar velocities behind it; in other words, the violet component of the emission line would be faint or invisible. On the other hand the red component, having a Doppler displacement in the opposite sense, would pass through unabsorbed and appear strong in the spectrum. With a collapsing atmosphere the reverse effect would occur, namely, a faint red component and a strong violet component.

While both these hypotheses, especially the latter, explain quite well the cyclic changes of the bright components in the normal Be-type stars such as  $\phi$  Persei, H.D. 20336, etc., they do not cover the case of  $\gamma$  Cassiopeiæ at the epoch when the bright violet components move across to the positions of the red components. Such an abrupt change in the normal procedure suggests that quite a new order of conditions is involved which at the present moment is not easy of explanation.

\* *Ap. J.*, 73, 94, 1931.

† *Nat. Ac. of Sciences*, 19, 44, 1933.



W. J. S. Lockyer, *The Variable Spectrum of  $\gamma$  Cassiopeiae.*



## DESCRIPTION OF PLATE 7

This plate illustrates the change of position of the single bright components of the hydrogen lines ( $H\gamma$  and  $H\delta$ ) from coincidence with the violet component (A and B) to coincidence with the red component (C and D).

The upper spectra A and C of each pair were taken on dates when both bright components were visible, namely, 1933 January 9 and 1932 July 20 respectively.

The lower of each pair B and D were taken on dates (1) *just after* the red component had disappeared, namely, 1934 April 9, and (2) *just before* the violet component appeared again, namely, 1934 November 2.

Each pair of spectra are so adjusted that all the lines other than hydrogen are made to coincide.

All the spectra indicate also the wide absorption band, midway between  $H\gamma$  and  $H\delta$ , which is generally such a conspicuous feature in many Be-type stars, as was pointed out by me in a previous paper (*M.N.*, **89**, 127, 1928).

The reproductions are made from direct enlargements from the original negatives magnified 5.5 times.

## THEORETICAL COLOUR TEMPERATURES.

*A. Pannekoek.*

At the meeting of the Society in 1934 April Mr. Greaves communicated the results of the Greenwich determinations of colour temperatures of stars, after the last comparisons with new absolute standards.\* The startling result of colour temperatures of  $18,000^\circ$  to  $20,000^\circ$  for A stars, though in accordance with other investigations, was at variance with the general belief that the radiation of the stars did not deviate much from black-body radiation corresponding to the effective temperature. Dr. Woolley shortly afterwards cleared up this contradiction and showed that we have no reason to expect the stars to radiate like black bodies.†

If we denote by  $k$  the mass absorption coefficient for a frequency  $\nu$ ; by  $\bar{k}$  the mean absorption coefficient for the total radiation (Rosseland mean), which determines the temperature increase with depth; by  $-k\rho dh = d\tau$  the optical depth for  $\nu$  radiation; by  $E(\nu)$  and  $E$  the black-body radiation for frequency  $\nu$  and integrated, then  $E$  increases with depth as  $E_0(1 + \frac{3}{2}\tau\bar{k}/k)$ , and  $E(\nu)$ , in the first approximation, increases as  $E_0(\nu)(1 + \frac{3}{2}c\tau\bar{k}/k)$ , where the coefficient  $c$  denotes ‡

$$c = \frac{d \log E(\nu)/dT}{d \log E/dT} = \frac{1}{4} \frac{c_2}{\lambda T} (1 - e^{-c_2/\lambda T})^{-1}.$$

\* *M.N.*, **94**, 488.

† *M.N.*, **94**, 713.

‡ Cf. Milne, *Handb. d. Astrophysik*, III, 1, p. 167.

Using the Eddington integrals of the radiation  $\int Id\omega = 4\pi\mathfrak{J}$ ;  $\int I \cos \vartheta d\omega = 4\pi H$  ( $\vartheta$  = angle of the ray with the outward vertical), the equations for radiation of each  $\nu$  of the continuous spectrum are

$$\frac{d\mathfrak{J}}{d\tau} = 3H, \quad \frac{dH}{d\tau} = \mathfrak{J} - E(\nu),$$

the solution of which (with surface condition  $\mathfrak{J} = 2H$ ) gives for the surface radiation:

$$H_0(\nu) = \frac{E_0(\nu)}{2 + \sqrt{3}} \left( 1 + \frac{\sqrt{3}}{2} c \frac{\bar{k}}{k} \right). \quad (1)$$

Hence the variation of  $H_0(\nu)$ , the background intensity in the integrated spectrum, depends on the variation of the ratio  $\bar{k}/k$  with wave-length.

The results for the continuous absorption coefficients  $k$ , derived at the Amsterdam Institute as part of a theoretical investigation of line intensities, may be used to derive the intensity distribution over the continuous spectrum and the corresponding colour temperature. Whereas for the details of the computations we must refer to a future publication of the whole investigation, we give here only the necessary foundations and chief results.

We assume the continuous absorption to be due for the lesser part to free-free transitions of electrons with ions, and for the main part to ionization; the formulæ of Kramers and Gaunt are used. In the case of hydrogen we write for the latter part:

$$k(\nu) = CT^{-5/2}\nu^{-3}SP; \quad S = x \sum_n \frac{2n}{(n^2 - \frac{1}{4})^2} e^{h\nu_n/kT}; \quad \nu_n = \nu_1/n^2,$$

where

$$C = \frac{8\pi^2 \sqrt{2\pi}}{3\sqrt{3}} \frac{\epsilon^{10}}{ch^3 m^{1/2} k^{5/2} m_H},$$

$P$  is the electron pressure,  $x$  the degree of ionization,  $\nu_1$  is the lowest,  $\nu_n$  some higher level or band edge, and the summation is extended over all  $n$ , up to  $\infty$ , for which the absorption bands cover this  $\nu$ , hence  $\nu_n < \nu$ .<sup>\*</sup> With increasing  $\nu$  the absorption coefficient  $k(\nu)$  continually decreases as  $\nu^{-3}$ , but then at each band edge jumps up to a higher value, because a term with the next lower value of  $n$  is added.

For the hotter stars we have only to consider the hydrogen absorption. Since the number of hydrogen atoms exceeds the metal atoms nearly a thousand times, the contribution of the latter to the absorption is entirely negligible. When, however, the temperature falls below 10,000° the ionization of hydrogen and with it the absorption coefficient rapidly decreases. Then the strongly ionized metal atoms replace hydrogen as the chief source of absorption, which is then 100 to 1000 times smaller than in the hotter atmospheres. With decreasing temperature the metals with highest ionization potential fall out one after another; but by the  $T$  factor the absorption

\* That the symbol  $k$  is used for Boltzmann's constant as well as for the absorption coefficient will cause no confusion.

coefficient gradually increases. If the metal atoms in absorbing radiation when ionizing behaved just like the hydrogen atoms, we should see conspicuous band edges (of *Mg*, *Na*, *K*) in the continuous spectra of class G and K stars. Since such edges are not observed, we have to assume that some smoothing process is at work with these atoms. Hence a smoothed absorption coefficient has been computed in the manner explained in my former paper \* by replacing the large number of irregularly situated band edges by a quasi-infinite number, each of which has  $d\nu$  instead of the finite  $\Delta\nu$  in Kramers' formula. Then the sum becomes an integral.

$$\sum_n \frac{p}{n^2} e^{-h(\nu_1 - \nu_n)/kT} \Delta\nu = \int_0^\nu e^{-h(\nu_1 - \nu_n)/kT} d\nu_n = \frac{kT}{h} e^{-h\nu_1/kT} (e^{h\nu/kT} - 1),$$

from which, after eliminating the number of neutral atoms by means of the ionization formula, we get

$$k(\nu) = C' T^{-3/2} \nu^{-3} x P (e^{h\nu/kT} - 1),$$

where

$$C' = \frac{4\sqrt{2\pi}}{3\sqrt{3}} \frac{\epsilon^6}{ch(mk)^{3/2} m_H}.$$

For the mean ionization  $x$  the ionization of each element in the Russell mixture had been computed separately.

For temperatures below 6000° this value increases with higher values of  $\nu$ , in consequence of the increasing number of lower, hence strongly occupied levels in these atoms. There is a domain of transition temperatures (between 10,000° and 6000°, depending on the pressure) where the absorption is a combined effect of hydrogen and metal atoms.

To this ionization part of the absorption the free-free transition part has been added, corrected for Gaunt coefficients; though these coefficients are rather uncertain this has not much influence upon the results. Then the Rosseland mean has been computed; corrections for the absorption by Fraunhofer lines were applied to the mean coefficient. Now by means of formula (1) the intensity of the continuous spectrum was computed for wave-lengths 4000 Å., 5000 Å., 6000 Å. Colour temperatures are derived by comparing intensity ratios within the visible spectrum, situated between the 2nd and the 3rd hydrogen edge. We will derive theoretical colour temperatures from the computed intensities, treating them by the same procedure as is followed with observed spectrum intensities.

In the table the highest lines give  $\log \bar{k}/k$  for these three wave-lengths as functions of the effective temperatures (for which the Russell values 5040/ $T = 0.2, 0.3, \dots$  were taken). The next lines show the logarithm of the factors  $(1 + \frac{1}{2}\sqrt{3} c\bar{k}/k)$ . They must be multiplied by  $E_0(\nu)$ , the black-body radiation corresponding to the surface temperature  $T_0$ , for

\* *M.N.*, **91**, 163. The derivation given there contains an error; by the omission of the weight  $p$  in the Boltzmann sum it was not seen that the denominator factor  $n^2$ , which means the same weight, was cancelled; hence the formula given there deviates from the present one.

which  $1/1.23$  times the effective temperature  $T_1$  was taken. In colour-temperature determinations the relative gradients are derived from  $\log(e^{c_2/\lambda T} - 1)^{-1}$ ; hence the relative intensities  $E_0(\nu)/E_1(\nu)$  are derived by this function, and by combining them with the above factors we get the relative intensity of the star spectrum to the black-body spectrum of the effective temperature. Since  $1/\lambda$  for the three wave-lengths is  $2.5$ ,  $2$ ,  $1\frac{2}{3}$ , we find the gradient  $\phi = (c_2/T)(1 - e^{-c_2/\lambda T})^{-1}$ , relative to the black-body gradient, by multiplying the differences of two consecutive lines by  $2 \times 2.30$  and  $3 \times 2.30$ . So the relative gradients  $\Delta\phi$  are found. Adding them to the gradients of the black-body radiation  $\phi(T_1)$  we find the apparent gradients  $\phi$ , as given in the last lines of the table.

| $5040^\circ/T$                                                               | 0.2                     | 0.3                     | 0.4                     | 0.5                     | 0.6                     | 0.7                     | 0.8                     | 0.9                     | 1.0                     | 1.2                     | 1.4                     |
|------------------------------------------------------------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| $\bar{k}$ 4000 Å.                                                            | -.73                    | -.36                    | +.09                    | +.38                    | +.48                    | +.33                    | +.02                    | -.19                    | -.35                    | -.71                    | -1.10                   |
| $\log \bar{k}$ 5000 Å.                                                       | -.98                    | -.65                    | -.20                    | +.09                    | +.19                    | +.19                    | +.15                    | +.06                    | -.03                    | -.26                    | -.53                    |
| $\bar{k}$ 6000 Å.                                                            | -1.22                   | -.89                    | -.44                    | -.15                    | -.05                    | -.01                    | +.15                    | +.16                    | +.12                    | -.02                    | -.19                    |
| $\log \left(1 + \frac{\sqrt{3} \bar{k}}{2 c \bar{k}}\right)$                 | +.032<br>+.016<br>+.009 | .090<br>.041<br>.022    | .256<br>.129<br>.070    | .463<br>.256<br>.146    | .583<br>.339<br>.199    | .521<br>.373<br>.237    | .360<br>.381<br>.340    | .278<br>.358<br>.371    | .227<br>.333<br>.374    | .133<br>.258<br>.338    | .069<br>.178<br>.285    |
| $\log \frac{E_0}{E_1} \left(1 + \frac{\sqrt{3} \bar{k}}{2 c \bar{k}}\right)$ | -.146<br>-.142<br>-.137 | -.145<br>-.159<br>-.156 | -.041<br>-.118<br>-.147 | +.101<br>-.041<br>-.110 | +.158<br>-.010<br>-.098 | +.022<br>-.029<br>-.104 | -.209<br>-.076<br>-.045 | -.362<br>-.155<br>-.059 | -.483<br>-.236<br>-.103 | -.719<br>-.423<br>-.231 | -.925<br>-.617<br>-.379 |
| $\Delta\phi$                                                                 | +.02<br>+.04            | -.06<br>+.02            | -.35<br>-.20            | -.65<br>-.48            | -.77<br>-.61            | -.23<br>-.52            | +.61<br>+.21            | +.95<br>+.66            | +1.14<br>+.92           | +1.36<br>+1.33          | +1.41<br>+1.64          |
| $\phi(T_1)$                                                                  | .79<br>.88              | 1.00<br>1.08            | 1.24<br>1.30            | 1.48<br>1.54            | 1.75<br>1.79            | 2.02<br>2.05            | 2.29<br>2.31            | 2.57<br>2.58            | 2.85<br>2.86            | 3.42<br>3.42            | 3.98<br>3.98            |
| $\phi$ app.                                                                  | .81<br>.92              | .94<br>1.10             | .89<br>1.10             | .83<br>1.06             | .98<br>1.18             | 1.79<br>1.53            | 2.90<br>2.52            | 3.52<br>3.24            | 3.99<br>3.78            | 4.78<br>4.75            | 5.39<br>5.62            |

The results are illustrated in fig. 1, where  $\phi$  for each of the wave-length intervals (denoted as 4500 and 5500) is given as a function of the effective temperature (abscissa  $c_2/T$ ). The value of  $\phi$  for black-body radiation is given by two curves, and the scales of colour temperature at the left-hand side correspond to these curves.

We see that when the effective temperature decreases from  $20,000^\circ$  to  $10,000^\circ$  the gradient in this part of the spectrum remains nearly constant, and the colour temperature is in the vicinity of  $20,000^\circ$  or  $18,000^\circ$ . This is just what was found by the Greenwich observers; their high colour temperatures for class A stars could have been foretold by our absorption coefficients. When, then, the effective temperature falls below  $10,000^\circ$  the colour temperature falls more rapidly, and below  $7000^\circ$  it becomes considerably lower than the effective temperature. It will be difficult to test this result, because in the spectra of stars of later type the wings of the numerous absorption lines overlap, and nowhere can the real continuous background of the spectrum be observed and measured.

For the high-temperature stars where the continuous background of the spectrum itself can be measured, the computation may be tested by observa-

tion. And here it must be remarked that, though in a first approximation theory is able to explain the Greenwich results, still there are notable discrepancies. At the high temperatures theoretical computation shows a flat wave, a minimum at  $T_1 = 16,000^\circ$ , a maximum at  $10,000^\circ$ . The Greenwich measures of relative colour show a continuous though at last slow increase from A5 and A0 through B5 to B0 (*cf.* fig. 10, p. 36, of the Greenwich

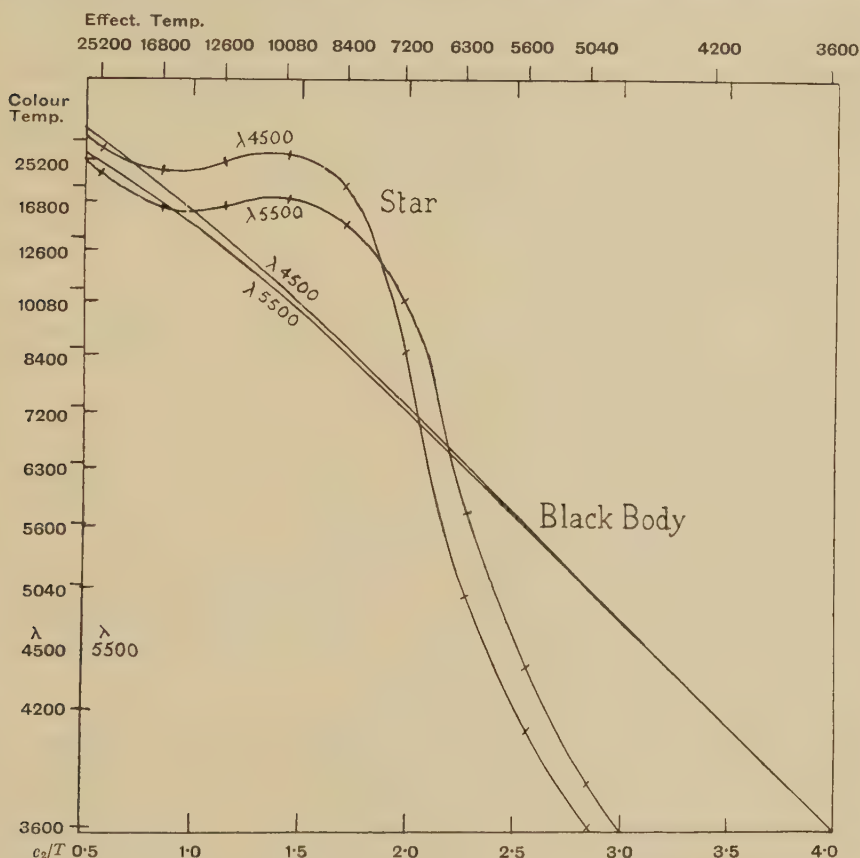


FIG. 1.

publication). At least this holds for the maximum values; the lower colour temperature for a number of B stars may safely be ascribed to space reddening, and cannot give an indication of really lower colour temperature. For the Sun, the difference between effective temperature ( $5800^\circ$ ) and colour temperature ( $6300^\circ$ ) is given by theory in the wrong direction; we should wish to have the curve displaced to the right-hand side. It must be left to further research to clear up these discrepancies and to see what changes have to be made in the absorption coefficients.

If we try to trace the origin of the strong systematic deviations of colour from effective temperature, we find it in the manner in which the mean absorption in a hydrogen atmosphere takes place. What is averaged in the Rosseland mean is the transparency. An energy stream driven by a tem-

perature gradient breaks through the different wave-lengths according to their transparency. In fig. 2 the absorption coefficient is given as a function of wave-length (both on a logarithmic scale; one vertical division is  $\log 10$ ), as well as the black-body energy, for three temperatures. The vertical distance of the energy curve above the  $k$  line is the logarithm of the energy passing through that wave-length. At  $25,200^\circ$  it is just above the Lyman band edge that most of the energy finds a passage. Hence the Rosseland mean corresponds with  $k$  in that corner and falls far below the value of  $k$  at  $4000 \text{ \AA}$ . For lower temperature the maximum of the energy curve is displaced towards larger wave-length; gradually the interval between the 1st and the 2nd band edge takes a smaller part, whereas an increasing part

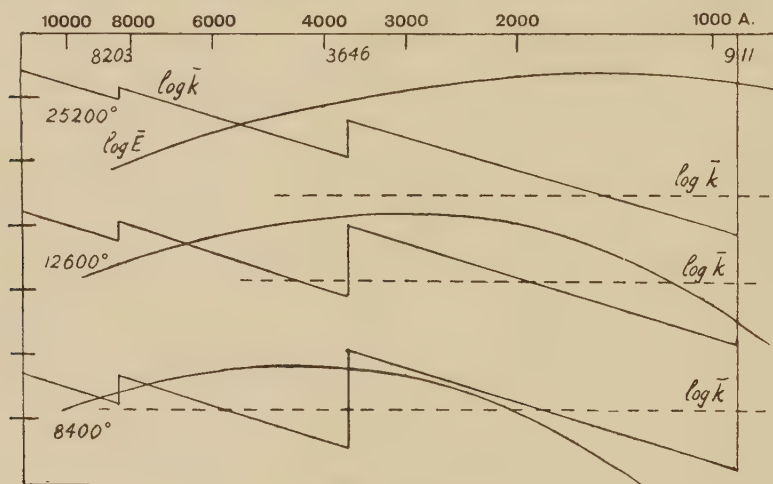


FIG. 2.

of the energy passes through the wave-lengths above the Balmer edge, the region of our measures. Then the mean  $\bar{k}$  rises at last above the  $k$  ( $4000$ ), and now the slope of  $k$  produces a strong relative gradient. With still lower temperatures the fourth (infra red) interval above the Paschen edge comes into action; but now hydrogen loses its influence upon the absorption coefficient. For high temperatures the mean absorption, which determines the increase of temperature with depth, is small compared with the visual absorption, which determines from what depth the emitted radiation is coming; hence the latter is radiation of little more than surface temperature. While for  $10,000^\circ$  the mean absorption is large compared with the visual absorption, we then look into deep layers of strongly increased temperature, producing a great intensity gradient in the visual part of the spectrum.

The limb darkening depends on the same absorption coefficients, since for oblique rays the intensity is  $E_0(1 + \frac{3}{2}c\bar{k}/k \cdot \cos \vartheta)$ . It is small at high temperatures; it is large and strongly variable with wave-length at  $10,000^\circ$ . For the three wave-lengths used we find the coefficient of darkening  $0.12$ ,  $0.06$ ,  $0.03$  at  $25,200^\circ$ ;  $0.78$ ,  $0.58$ ,  $0.41$  at  $10,080^\circ$ .

Colour temperature is not, like effective temperature, a quality or physical

datum of a star. It is a number expressing special observational arrangements, and it varies with the wave-lengths used. Its qualitative behaviour depends on the behaviour of the absorption coefficient in the special interval between the Balmer and the Paschen edge of hydrogen, relative to the mean absorption coefficient. Since the colour index, found from photographic and visual magnitudes, and the effective wave-length, determined by the maximum of photographic intensity between 4000 Å. and 5000 Å., depend on the intensity variation in the same interval, there must be a close correlation between these three data: colour temperature, colour index and effective wave-length. Their correlation, however, with the effective temperature of the star, is less and irregular. The figure of the curve in fig. 1 must have some bearing upon the statistical distribution of star colours.

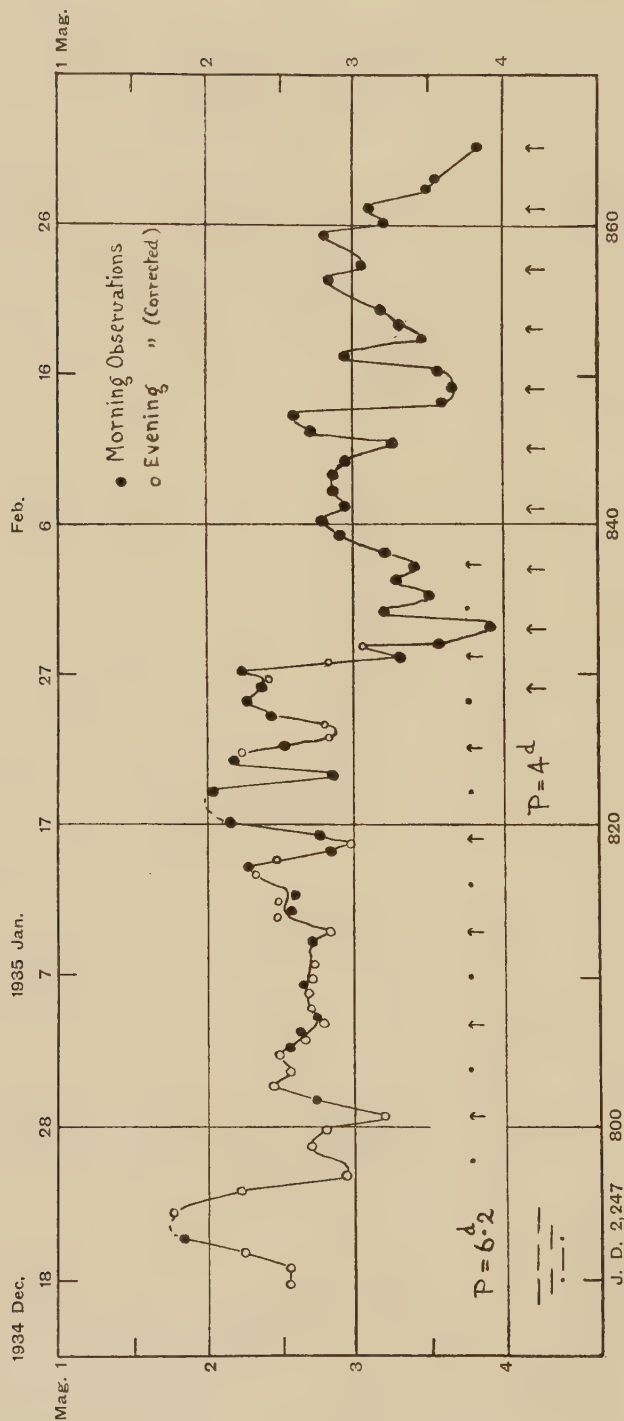
## NOVA HERCULIS 1934: SOME PECULIARITIES OF THE LIGHT-CURVE.

*P. M. Ryves.*

From December 18 onwards observations of Nova Herculis have been made on every available occasion, 12 nights in December, 30 in January and 27 in February. The observer has received no information later than December 31 regarding results obtained elsewhere, so that the conclusions arrived at here are independent of the work of other investigators. Although the final light-curve will no doubt be obtained from a discussion of a large number of observations from many sources a series of observations all made by one person has just this advantage, it is free from certain personal differences and discrepancies which cause much trouble in the reduction and analysis and which tend to mask the smaller secondary variations.

On several dates in January observations were obtained both in the evening and the morning. In the case of the morning observations the nova was high in the sky, but in the evening the star was low and the observations are affected by atmospheric extinction. A comparison of the morning and evening estimates enabled a general correction to be applied to the latter. After the end of January evening observations became impossible in this latitude.

An examination of the unipersonal light-curve reveals some interesting characteristics. After the principal maximum, 1<sup>m</sup>.8 (Dec. 21-22), the light declined rapidly to about third magnitude (Dec. 25-26), and this was followed by a slow general recovery during which the light fluctuated and minima took place at intervals of about six days (6.2), there being indications of small secondary minima between the deeper ones. About the middle of January some larger and rapid fluctuations occurred which upset or obscured the 6-day period, the 4th minimum being apparently "frustrated" by a sudden rise, though the 5th and 6th are still traceable. By the end



*Nova Herculis*: Unipersonal light-curve based upon 83 observations made on 72 nights, at Zaragoza, Spain. After December, the evening observations, shown by circles, are less reliable than the morning. Note that the short term periodic variation is best defined when the nova is slowly increasing in brightness and is liable to be upset by a sudden decrease or large oscillations.

Scale : 1 mag. = 10 days = 2 cm.

of the month (January 27) the star had recuperated about half a magnitude when a sudden fall of nearly two magnitudes took place, and this seems to have finally upset the 6-day periodicity. Another climb up, not so slow as the previous one, now begins, and the general brightness of the nova increases about  $0^m.7$  in 15 days. Minima are now taking place as before, but at intervals of 4 days instead of 6, and there is no clear indication of secondaries between them. The two periods, 4 and 6 days, seem to overlap the very shallow minimum of January 26, though doubtful, may be the beginning of the 4-day periodicity, whilst the curious hitch on January 28 and the minimum shown on February 1 seem to be the continuation of the 6-day period.

Between February 13 and 14 a third fall takes place, but not so considerable as the preceding ones, and again the slow recovery begins. This smaller fall has not upset the 4-day period, so far as can be ascertained, in spite of the bad weather prevailing at the end of the period under discussion. The last climb-up, also about  $0.7$  magnitude, is still a little steeper than the previous ones and also shorter, and this suggests that the feature will soon disappear.

An outstanding point of interest is the remarkable similarity between the light-curve of Nova Herculis, as revealed by these observations and that of V Sagittæ.\* Rapid fluctuations with an amplitude of one magnitude or more, a sudden and considerable fall followed by a slow crawl-up, secondary periodic or semi-periodic variations superimposed on the other light changes, and finally the fact that these latter variations, or rather minima, are very diverse in form and depth and yet occur with remarkable punctuality at their appointed times—all these are characteristics common to both stars. Whether Nova Herculis will show anything corresponding to the 18-month period of V Sagittæ remains to be seen.

The latter star is now due to rise to one of its long-period maxima which are followed by the more remarkable and rapid fluctuations, and it is very desirable that its spectrum should be photographed at different phases, as is being done in the case of Nova Herculis. A comparison of the results might lead to important conclusions.

*Zaragoza, Spain :*  
1935 March 5.

\* *M.N.*, 92, 715, and Plate 15, 1932.

## THE LIGHT-CURVE AND THE VISUAL SPECTRUM OF NOVA HERCULIS 1934.

*Arthur Beer, Ph.D.*

The first week of 1935 April brought a very striking change in the life-history of this nova, peculiar in so many respects. Within a few days the light dropped more than five magnitudes; on March 31 the star was brighter than 4<sup>m</sup>.5; on April 2, 5<sup>m</sup>.3; on April 3, 0<sup>h</sup>, it was 6<sup>m</sup>.2, and 24 hours later already 7<sup>m</sup>.6; April 5, 8<sup>m</sup>.4; April 10, 9<sup>m</sup>.4; and on April 23 the 12th magnitude was reached; April 27, 12<sup>m</sup>.8. It is of interest to note that no other nova, with perhaps the single exception of Nova Aurigæ 1891 (*T*), has behaved in a similar manner, as may be seen on inspecting the light-curves.\* Nova Pictoris 1925,† whose light-curve showed a certain resemblance to that of Nova Herculis during the first three months, was in its later stages characterized by a drop about only a hundredth as rapid as that of the present nova.

A preliminary light-curve, here reproduced (fig. 1 and Plate 8), has been drawn for use during the spectroscopic work made at the Solar Physics Observatory at Cambridge since the discovery.

It is based on 4562 observations from various sources ‡ (including daily eye estimates made in Cambridge), which came to hand before the end of April. All single magnitudes were plotted against time, without using mean values over short periods, in order not to eliminate real fluctuations. The observations have been reduced to the Harvard system but have received no further modifications.

Accurate corrections for extinction, particularly important for the numerous low-altitude observations, have not always been applied by the observers. We may also expect physiological colour effects, introduced by the use of instruments of different light-gathering power; also the colour-sensitiveness of the eyes of different observers should be allowed for. As particulars of the exact conditions are still lacking, the curve has been fitted best to the points plotted from the uncorrected observations. It was of

\* F. J. M. Stratton, *Handbuch d. Astrophysik*, vi/2, p. 263, 1925; L. Campbell, *H.A.*, 81, 113, 1919; *H.A.*, 84, 153, 1920; *H.A.*, 84, 205, 1923.

† H. Spencer Jones, *Cape Ann.*, x, 9; p. 11, i; 1931.

‡ *A.N.*, Nos. 6078-6103, 1934-35; *Circulaires I.A.U.*, Nos. 501-534, Observatory, Copenhagen; *Journ. B.A.A.*, 45, 145, 1935; *Zirkular Nr. 1, Mittelmeerstation d. Wiener Univ.-Sternwarte* (K. Graff); *Bulletin Association Française d'Observateurs d'Etoiles Variables*, iii, iv, 1934; H. Shapley: *A.N.*, 6082, p. 156 (*H.A. Card* 321); *Gazette Astronomique* (Anvers), 22, 17, 1935; *Sternberg State Astron. Inst. Moscow*, Card 3; *P.A.*, 43, 114 and 173, 1935; *Journ. R.A.S. Canada*, 29, 15 and 49, 1935; *Obs. Astr. Kharkow Bull.*, No. 1, 1935; *Astron. Circulars Tashkent Observatory*, Nos. 36, 38, 1935.

course found necessary to disregard those observations which deviate too greatly from the majority.

Concerning the first light increase, we may note that on the evening of 1934 December 12 at 21 hours, according to the observation of the Director of the Armagh Observatory, the nova seemed fainter than  $6^m.0$ .<sup>\*</sup> This agrees with the observation of D. McLaughlin,<sup>†</sup> that before Dec. 13 no star brighter than 2nd magnitude was visible in this region, but possibly a star as faint as  $3.0$  would have escaped notice. Prentice's discovery on December 12 showed the star  $3^m.4$ , and Steavenson<sup>‡</sup> estimated it on December 13 between 12 and 15.30 G.C.T. to be brighter than  $3^m.0$ , perhaps

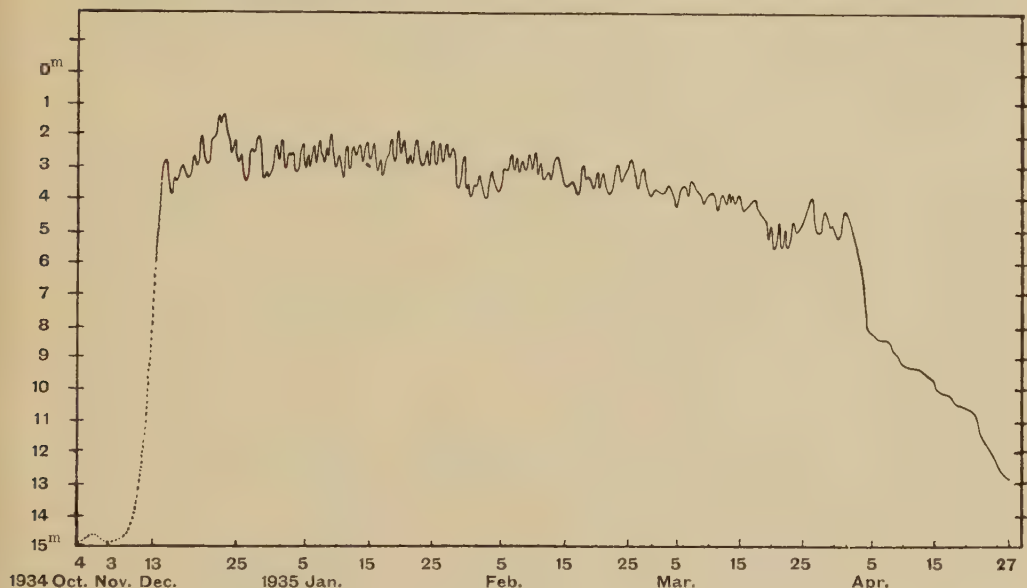


FIG. 1.—Light-curve of Nova Herculis (1934 December 13 to 1935 April 27).

about  $2^m.5$ . From this date onwards the light-curve has been fixed every day by a large number of observations.

Acknowledgment is due to M. Beyer, who has kindly contributed accurate unpublished photometric measures, made at his observatory at Hamburg, which have especially helped in the drawing of the last part of the light-curve.—Further, Mr. W. M. Lindley has been kind enough to communicate unpublished estimates of the Variable Star Section of the British Astronomical Association; and also Dr. W. H. Steavenson has kindly sent a complete list of his observations.

As our figures show, the general appearance of the light-curve can be characterized as follows:—

The discovery was made after a steep increase, probably within a day, of at least two magnitudes. A slower fluctuating increase followed between

<sup>\*</sup> *The Observatory*, 58, 76, 1935; *M.N.*, 95, 356, 1935.

<sup>†</sup> D. McLaughlin, *P.A.*, 43, 93, 1935.

<sup>‡</sup> *Journ. of the B.A.A.*, 45, 147, 1935.

December 13 and December 21. The principal maximum, perhaps a double one, was on Dec. 22 and 23 with the chief peak  $1^m.3$  on Dec. 23.1.

The star then decreased till Dec. 26.5,  $3^m.5$ ; maximum Dec. 28.3,  $2^m.0$ ; again minimum Dec. 30.0,  $3^m.4$ . From then onwards during a whole month the light remained, fluctuating, within the range of one magnitude, that is to say between  $2^m$  and  $3^m$ . During this month the star reached its second maximum of  $1^m.8$ , on Dec. 19.4. Between January 28 and February 6 some disturbance caused a deep shallow minimum on Feb. 2.5,  $3^m.9$ . Then for the second week of February it was again brighter than  $3^m$ , probably with small fluctuations similar to those during January. Between February 12 and February 27 we note larger fluctuations; and on the last-mentioned date the star was for the last time of the third magnitude; from then onwards a continuous drop occurred till the end of March, covering about two magnitudes, and this was followed in April by the rapid fading already mentioned.

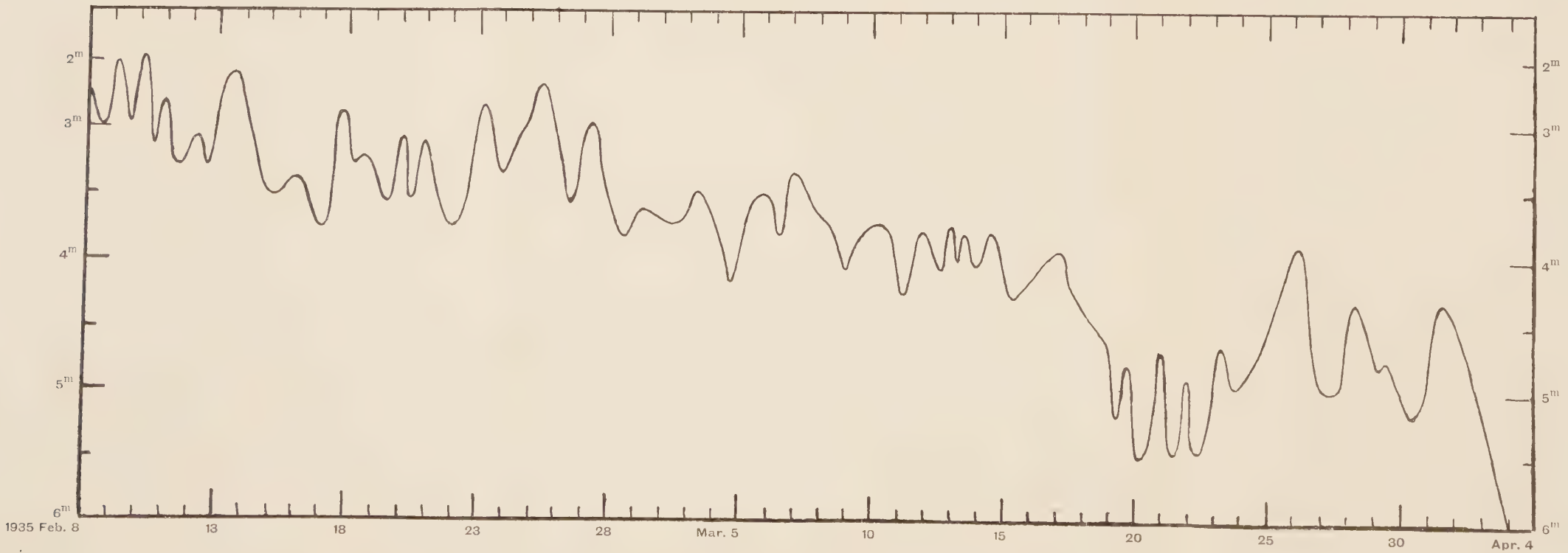
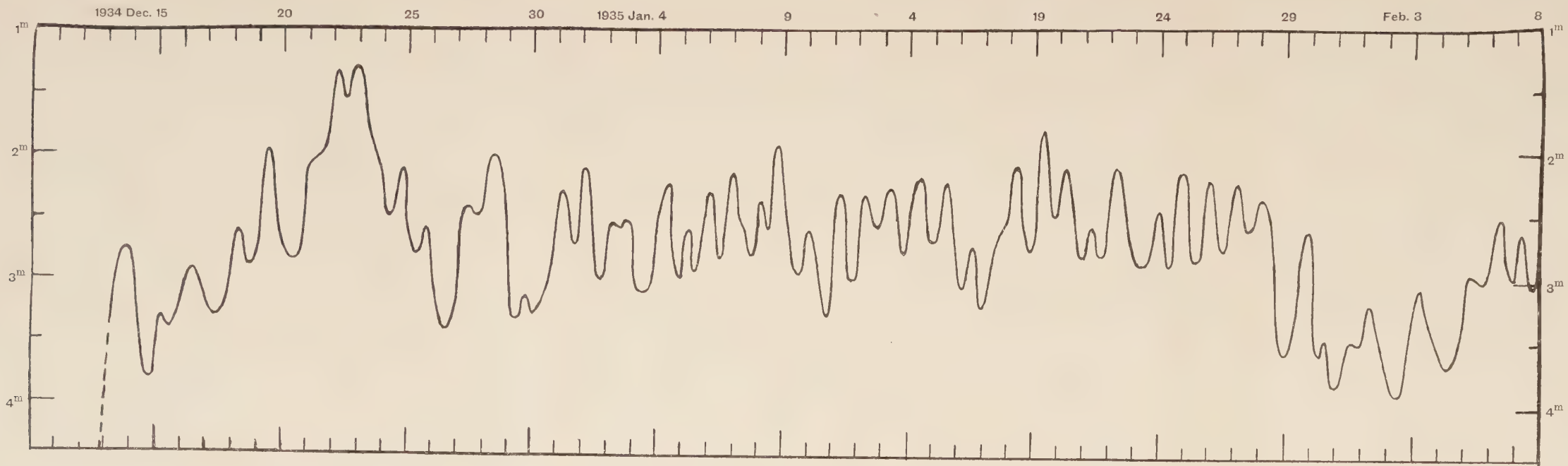
It seems unnecessary in view of the curves to state in detail the large number of definite maxima and minima of the nova, which are quite distinct in the course of its light history. Some of the smaller short-period fluctuations are of course affected by the scattering of the points due to observational errors. But the majority are probably real, and seem to indicate that, especially during January, the star was remarkably unsteady and experienced fluctuations of the order of half a magnitude in very short intervals.

The spectral changes accompanying the light variations have been studied at the Solar Physics Observatory, where some 400 plates have been secured ranging from the extreme ultra-violet to  $H\alpha$  and a little beyond. In this note we are concerned only with some of the 112 plates obtained by the writer with the Huggins 15-inch refractor, and covering the range  $H\beta$  to  $H\alpha$ . The exposures varied from 30 minutes in December and January to 10 hours in April, the last plate being spread over two nights, April 10-12.

Comparison plates were taken with standardized wedge spectra, but the detailed photometric study must be left till later. For the present only rough estimates of intensity changes are available. The lines 6300 and 6364 of [OI] \* have strengthened relatively to all other lines of the spectrum with the possible exception of  $H\alpha$ , and are next to  $H\alpha$  in strength on the plates secured after the great drop in brightness in early April.

It is to be noted that after this drop of brightness the forbidden lines 5577 [OI] and 5755 [NII], which had both weakened considerably, have once again been very much strengthened. Along with the  $FeII$  lines the [OI] lines have the violet maxima very much stronger than the red maxima. They now show a Doppler displacement to the violet corresponding to a velocity of  $-440$  km./sec. which makes the identification of some of the other emission lines somewhat difficult. In the table below, pp. 541-548, the separate numbered groups refer to bands of changing composition, frequently blended and cut across by absorption lines.

\* F. J. M. Stratton and Arthur Beer, "On Neutral Oxygen and Carbon in Nova Herculis 1934," *M.N.*, 95, 432, 1935.



Light-curve of Nova Herculis 1934-35.



In preparing this table all available plates were inspected after a special investigation had been made of the plate of 1935 January 4.31, which shows the majority of the lines present; for further reference see the remarks at the end. The hypersensitized plates showing lines on the red side of *Ha* up to  $\lambda$  7330 were taken only during the period 1935 March 15-25.

The headings of the successive columns in the table denote :

- (1) and (2) : Wave-lengths and origin of lines clearly present.  
 (3) and (4) : Laboratory-intensities and multiplet classification.\*  
 (5) and (6) : Wave-lengths and origin of other lines possibly present in the same bright band.  
 (7) and (8) : Laboratory-intensities and multiplet classification.  
 (9) : No. referring to the remarks (to be found at the end of the table) which concern identification and especially the behaviour of lines during the observation period.  
 Here also reference is made for some lines to the spectra of Nova Pictoris, †  $\alpha$  Cygni, ‡  $\alpha$  Persei. §

When a band may be traced to a less certain origin, all possible lines are shown in the columns (1) to (4). If an identification seems altogether doubtful, it is put in the columns (5) to (8) only, the measured centre of the bright band ( $\pm$ ) being entered in column (1), which otherwise contains only laboratory wave-lengths.

*Emission Bands identified in the Visual Spectrum of Nova Herculis*

| (1)<br>$\lambda$   | (2)<br>Origin              | (3)<br><i>I</i> | (4)<br>Multiplet                                                                               | (5)<br>$\lambda$              | (6)<br>Origin              | (7)<br><i>I</i> | (8)<br>Multiplet                                                                                                                               | (9)<br>Re-<br>marks |
|--------------------|----------------------------|-----------------|------------------------------------------------------------------------------------------------|-------------------------------|----------------------------|-----------------|------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|
| 4861.34            | <i>H<math>\beta</math></i> | ...             | $2^2P^0 - 4^2D$                                                                                | 4861.03<br>4864.32<br>4871.58 | OII<br><i>Cr</i> II<br>OII | 3<br>3<br>3     | $f^2P_{\frac{1}{2}}^0 - w^2D_{1\frac{1}{2}}$<br>$a^4F_{2\frac{1}{2}} - z^4F_{2\frac{1}{2}}^0$<br>$f^2P_{1\frac{1}{2}}^0 - w^2D_{2\frac{1}{2}}$ | 1                   |
| 4890.93<br>4906.88 | OII<br>OII                 | 4<br>5          | $e^4S_{1\frac{1}{2}}^0 - x^4P_{\frac{1}{2}}$<br>$e^4S_{1\frac{1}{2}}^0 - x^4P_{1\frac{1}{2}}$  | 4911.20                       | <i>Ti</i> II               | 0               | $a^2D_{2\frac{1}{2}} - y^2P_{1\frac{1}{2}}^0$                                                                                                  | 2                   |
| 4923.93<br>4924.60 | <i>Fe</i> II<br>OII        | 20<br>6         | $a^6S_{2\frac{1}{2}} - z^6P_{1\frac{1}{2}}^0$<br>$e^4S_{1\frac{1}{2}}^0 - x^4P_{2\frac{1}{2}}$ | 4921.93                       | <i>He</i> I                | 4               | $2^1P_1^0 - 4^1D_2$                                                                                                                            | 3                   |
| 4932.00<br>4934.04 | CI<br><i>Ba</i> II         | 5<br>(700)      | $3s^1P_1^0 - 4p^1P_1$<br>$6^2S_{\frac{1}{2}} - 6^2P_{\frac{1}{2}}^0$                           | ...                           | ...                        | ...             | ...                                                                                                                                            | 4                   |

\* C. E. Moore, *A Multiplet Table of Astrophysical Interest*, Princeton, 1933;

I. S. Bowen, *Ap. J.*, **81**, 12-13, 1935.

† H. Spencer Jones, *Cape Ann.*, **10**, 9, 18, etc., 1931.

‡ W. H. Wright, *L.O.B.*, **10**, 108, 1921.

§ Th. Dunham, jun., *Princeton Contr.*, No. 9, 1929.

| (1)<br>$\lambda$ | (2)<br>Origin | (3)<br>$I$ | (4)<br>Multiplet                              | (5)<br>$\lambda$ | (6)<br>Origin | (7)<br>$I$ | (8)<br>Multiplet                                | (9)<br>Re-<br>marks |
|------------------|---------------|------------|-----------------------------------------------|------------------|---------------|------------|-------------------------------------------------|---------------------|
| 4941.12          | OII           | 5          | $e^2P_{\frac{1}{2}}^0 - x^2D_{1\frac{1}{2}}$  | ...              | ...           | ...        | ...                                             | 5                   |
| 4943.06          | OII           | 7          | $e^2P_{1\frac{1}{2}}^0 - x^2D_{2\frac{1}{2}}$ |                  |               |            |                                                 |                     |
| 4955.78          | OII           | 3          | $e^2P_{1\frac{1}{2}}^0 - x^2D_{1\frac{1}{2}}$ |                  |               |            |                                                 |                     |
| 4987.38          | NII           | 4          | $3p^3S_1 - 3d^3P_0^0$                         | 4986.82          | LaII          | (100)      | $a^1D_2 - z^3D_1^0$                             | 6                   |
| 4994.36          | NII           | 6          | $3p^3S_1 - 3d^3P_1^0$                         |                  |               |            |                                                 |                     |
| 5001.13          | NII           | 7          | $3p^3D_1 - 3d^3F_2^0$                         | 5001.48          | CaII          | 2          | $5^2P_{\frac{1}{2}}^0 - 6^2D_{1\frac{1}{2}}$    | 7                   |
| 5001.47          | NII           | 8          | $3p^3D_2 - 3d^3F_3^0$                         | 5010.22          | TiII          | ...        | $c^2D_{1\frac{1}{2}} - x^2D_{1\frac{1}{2}}^0$   |                     |
| 5005.14          | NII           | 10         | $3p^3D_3 - 3d^3F_4^0$                         |                  |               |            |                                                 |                     |
| 5007.32          | NII           | 7          | $3p^3S_1 - 3d^3P_2^0$                         |                  |               |            |                                                 |                     |
| 5010.62          | NII           | 6          | $3s^3P_0^0 - 3p^3S_1$                         |                  |               |            |                                                 |                     |
| 5016.39          | NII           | 5          | $3p^3D_2 - 3d^3F_2^0$                         | 5013.70          | TiII          | ...        | $b^2D_{2\frac{1}{2}} - z^4D_{2\frac{1}{2}}^0$   | 8                   |
| 5018.45          | FeII          | 20         | $a^6S_{2\frac{1}{2}} - z^6P_{2\frac{1}{2}}^0$ | 5015.68          | HeI           | 6          | $2^1S_0 - 3^1P_1^0$                             |                     |
| 5025.66          | NII           | 6          | $3p^3D_3 - 3d^3F_3^0$                         | 5019.98          | CaII          | 3          | $5^2P_{1\frac{1}{2}}^0 - 6^2D_{2\frac{1}{2}}$   |                     |
|                  |               |            |                                               | 5021.15          | CaII          | 1          | $5^2P_{1\frac{1}{2}}^0 - 6^2D_{1\frac{1}{2}}$   |                     |
| 5031.03          | ScII          | 4          | $b^1D_2 - z^1P_1^0$                           | 5047.74          | HeI           | 2          | $2^1P_1^0 - 4^1S_0$                             | 9                   |
| 5039.07          | CI            | 3          | $3s^3P_1^0 - 4p^3D_2$                         |                  |               |            |                                                 |                     |
| 5041.13          | SiII          | 8          | $4^2P_{\frac{1}{2}}^0 - 4^2D_{1\frac{1}{2}}$  |                  |               |            |                                                 |                     |
| 5041.66          | CI            | 3          | $3s^3P_2^0 - 4p^3D_3$                         |                  |               |            |                                                 |                     |
| 5045.10          | NII           | 8          | $3s^3P_2^0 - 3p^3S_1$                         |                  |               |            |                                                 |                     |
| 5052.16          | CI            | 6          | $3s^1P_1^0 - 4p^1S_0$                         | 5056.35          | SiII          | 2          | $4^2P_{1\frac{1}{2}}^0 - 4^2D_{1\frac{1}{2}}$   | 10                  |
| 5056.17          | SiII          | 10         | $4^2P_{1\frac{1}{2}}^0 - 4^2D_{2\frac{1}{2}}$ |                  |               |            |                                                 |                     |
| 5072.30          | TiII          | 2          | $c^2D_{2\frac{1}{2}} - x^2D_{2\frac{1}{2}}^0$ | ...              | ...           | ...        | ...                                             | 11                  |
| 5103. $\pm$      | ...           | ...        | ...                                           | 5100.71          | ...           | ...        | ...                                             | 12                  |
| 5129.16          | TiII          | 1          | $b^2G_{4\frac{1}{2}} - z^2G_{4\frac{1}{2}}^0$ | 5132.96          | CII           | 3          | $3s^4P_{\frac{1}{2}}^0 - 3p^4P_{1\frac{1}{2}}$  | 13                  |
|                  |               |            |                                               | 5133.29          | CII           | 2          | $3s^4P_{1\frac{1}{2}}^0 - 3p^4P_{2\frac{1}{2}}$ |                     |
| 5154.08          | TiII          | 0          | $b^2D_{1\frac{1}{2}} - z^2D_{2\frac{1}{2}}^0$ | 5143.49          | CII           | 2          | $3s^4P_{1\frac{1}{2}}^0 - 3p^4P_{\frac{1}{2}}$  | 14                  |
| 5158.05          | [FeII]        | ...        | $a^4F_{3\frac{1}{2}} - b^4P_{2\frac{1}{2}}$   | 5145.16          | CII           | 5          | $3s^4P_{2\frac{1}{2}}^0 - 3p^4P_{2\frac{1}{2}}$ |                     |
|                  |               |            |                                               | 5146.06          | OI            | 5          | $3^3P - 9^3S_1^0$                               |                     |
|                  |               |            |                                               | 5151.08          | CII           | 3          | $3s^4P_{2\frac{1}{2}}^0 - 3p^4P_{1\frac{1}{2}}$ |                     |
|                  |               |            |                                               | 5160.02          | OII           | 4          | $e^2P_{\frac{1}{2}}^0 - x^2P_{\frac{1}{2}}$     |                     |
| 5169.05          | FeII          | 20         | $a^6S_{2\frac{1}{2}} - z^6P_{3\frac{1}{2}}^0$ | ...              | ...           | ...        | ...                                             | 15                  |
| 5185.91          | TiII          | 2          | $b^2G_{3\frac{1}{2}} - z^2G_{3\frac{1}{2}}^0$ | 5190.56          | OII           | 3          | $e^2P_{\frac{1}{2}}^0 - x^2P_{1\frac{1}{2}}$    | 16                  |
| 5188.70          | TiII          | 6          | $b^2D_{2\frac{1}{2}} - z^2D_{2\frac{1}{2}}^0$ |                  |               |            |                                                 |                     |
| 5197.58          | FeII          | 4          | $a^4G_{2\frac{1}{2}} - z^4F_{1\frac{1}{2}}^0$ | 5206.73          | OII           | 5          | $e^2P_{1\frac{1}{2}}^0 - x^2P_{1\frac{1}{2}}$   | 17                  |
| 5202.51          | SiII          | 3          |                                               |                  |               |            |                                                 |                     |
| 5211.54          | TiII          | 0          | $b^2F_{3\frac{1}{2}} - y^2F_{3\frac{1}{2}}^0$ | ...              | ...           | ...        | ...                                             | 18                  |

| (1)<br>$\lambda$ | (2)<br>Origin | (3)<br>$I$ | (4)<br>Multiplet                              | (5)<br>$\lambda$              | (6)<br>Origin      | (7)<br>$I$      | (8)<br>Multiplet                                                                                                    | (9)<br>Re-<br>marks |
|------------------|---------------|------------|-----------------------------------------------|-------------------------------|--------------------|-----------------|---------------------------------------------------------------------------------------------------------------------|---------------------|
| 5226.55          | TiII          | 5          | $b^2D_{1\frac{1}{2}} - z^2D_{1\frac{1}{2}}^0$ | ...                           | ...                | ...             | ...                                                                                                                 | 19                  |
| 5234.63          | FeII          | 4          | $a^4G_{3\frac{1}{2}} - z^4F_{2\frac{1}{2}}^0$ | 5237.33<br>5239.82            | CrII<br>ScII       | 2<br>2          | $b^4F_{4\frac{1}{2}} - z^4F_{4\frac{1}{2}}^0$<br>$a^1S_0 - z^1P_1^0$                                                | 20                  |
| 5256.93          | FeII          | ...        | $a^6S_{2\frac{1}{2}} - z^6F_{2\frac{1}{2}}^0$ | ...                           | ...                | ...             | ...                                                                                                                 | 21                  |
| 5262.15          | TiII          | 0          | $b^2D_{2\frac{1}{2}} - z^2D_{1\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5264.81          | FeII          | I          | $a^4G_{2\frac{1}{2}} - z^4D_{1\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5268.62          | TiII          | I          | $b^2F_{2\frac{1}{2}} - y^2F_{2\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5276.00          | FeII          | 5          | $a^4G_{4\frac{1}{2}} - z^4F_{3\frac{1}{2}}^0$ | 5275.08                       | OI                 | 4               | $3^3P - 7^3D^0$                                                                                                     | 22                  |
| 5273.39          | [FeII]        | ...        | $a^4F_{4\frac{1}{2}} - b^4P_{2\frac{1}{2}}^0$ | 5284.11<br>5285.34<br>5299.00 | FeII<br>CaII<br>OI | ...<br>...<br>5 | $a^6S_{2\frac{1}{2}} - z^6F_{3\frac{1}{2}}^0$<br>$5^2P_{\frac{1}{2}}^0 - 7^2S_{\frac{1}{2}}^0$<br>$3^3P - 8^3S_1^0$ |                     |
| 5305.87          | CrII          | I          | $a^4P_{2\frac{1}{2}} - y^4P_{2\frac{1}{2}}^0$ | 5307.30                       | CaII               | ...             | $5P_{1\frac{1}{2}}^0 - 7^2S_{\frac{1}{2}}^0$                                                                        | 23                  |
| 5308.43          | CrII          | I          | $b^4F_{3\frac{1}{2}} - z^4F_{2\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5310.69          | CrII          | I          | $b^4F_{1\frac{1}{2}} - z^4F_{2\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5313.59          | CrII          | I          | $b^4F_{2\frac{1}{2}} - z^4F_{2\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5316.62          | FeII          | 8          | $a^4G_{5\frac{1}{2}} - z^4F_{4\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5318.36          | ScII          | 3          | $b^1D_2 - z^3P_1^0$                           |                               |                    |                 |                                                                                                                     |                     |
| 5325.56          | FeII          | 0          | $a^4G_{3\frac{1}{2}} - z^4F_{3\frac{1}{2}}^0$ | 5328.98                       | OI                 | 6               | $3^5P_1 - 5^5D^0$                                                                                                   | 24                  |
| 5330.66          | OI            | 10         | $3^5P_3 - 5^5D^0$                             | 5329.59                       | OI                 | 7               | $3^5P_2 - 5^5D^0$                                                                                                   |                     |
| 5334.87          | CrII          | I          | $b^4F_{1\frac{1}{2}} - z^4F_{1\frac{1}{2}}^0$ | 5339.29                       | CaII               | ...             | $4^2F^0 - 7^2G$                                                                                                     |                     |
| 5337.76          | CrII          | I          | $b^4F_{2\frac{1}{2}} - z^4F_{1\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5357.20          | ScII          | 2          | $a^3P_0 - z^1P_1^0$                           | ...                           | ...                | ...             | ...                                                                                                                 | 25                  |
| 5362.87          | FeII          | ...        | $a^4G_{4\frac{1}{2}} - z^4D_{3\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5381.03          | TiII          | I          | $b^2D_{1\frac{1}{2}} - z^2F_{2\frac{1}{2}}^0$ | 5380.24                       | CI                 | 8               | $3s^1P_1^0 - 4p^1D_2$                                                                                               | 26                  |
| 5407.62          | CrII          | I          | $a^4P_{2\frac{1}{2}} - z^4P_{2\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5418.78          | TiII          | 0          | $b^2D_{2\frac{1}{2}} - z^2F_{2\frac{1}{2}}^0$ | ...                           | ...                | ...             | ...                                                                                                                 | 27                  |
| 5420.93          | CrII          | I          | $a^4P_{1\frac{1}{2}} - z^4P_{\frac{1}{2}}^0$  |                               |                    |                 |                                                                                                                     |                     |
| 5436.83          | OI            | 8          | $3^5P_3 - 6^5S_2^0$                           | 5435.16<br>5435.78            | OI<br>OI           | 5<br>6          | $3^5P_1 - 6^5S_2^0$<br>$3^5P_2 - 6^5S_2^0$                                                                          | 28                  |
| 5456.11          | SiII          | 2          | ...                                           | ...                           | ...                | ...             | ...                                                                                                                 | 29                  |
| 5468.92          | SiII          | 2          |                                               |                               |                    |                 |                                                                                                                     |                     |
| 5478.38          | CrII          | I          | $b^4G_{5\frac{1}{2}} - z^4F_{4\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5502.09          | CrII          | I          | $b^4G_{4\frac{1}{2}} - z^4F_{3\frac{1}{2}}^0$ | 5512.71                       | OI                 | 5               | $3^3P - 6^3D^0$                                                                                                     | 30                  |
| 5503.24          | CrII          | I          | $b^4G_{2\frac{1}{2}} - z^4F_{1\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5508.64          | CrII          | I          | $b^4G_{3\frac{1}{2}} - z^4F_{2\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5510.74          | CrII          | I          | $a^4P_{2\frac{1}{2}} - z^4P_{1\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |
| 5526.82          | ScII          | (75)       | $a^1G_4 - z^1F_3^0$                           | ...                           | ...                | ...             | ...                                                                                                                 | 31                  |
| 5534.85          | FeII          | ...        | $b^2H_{5\frac{1}{2}} - z^4F_{4\frac{1}{2}}^0$ |                               |                    |                 |                                                                                                                     |                     |

| (1)<br>$\lambda$ | (2)<br>Origin | (3)<br>$I$ | (4)<br>Multiplet                                | (5)<br>$\lambda$   | (6)<br>Origin     | (7)<br>$I$ | (8)<br>Multiplet                                | (9)<br>Re-<br>marks |
|------------------|---------------|------------|-------------------------------------------------|--------------------|-------------------|------------|-------------------------------------------------|---------------------|
| 5554.94          | OI            | 6          | $3^3P - 7^3S_1^0$                               | ...                | ...               | ...        | ...                                             | 32                  |
| 5577.34          | [OI]          | ...        | $2p^1D_2 - 2p^2S_0$                             | 5576.61            | SiII              | 1          | $4s^4P_{1\frac{1}{2}}^0 - 4p^4S_{1\frac{1}{2}}$ | 33                  |
| 5627.51          | $\odot +$     | ...        | ...                                             | ...                | ...               | ...        | ...                                             | 34                  |
| 5639.49          | SiII          | 2          | $4s^4P_{2\frac{1}{2}}^0 - 4p^4S_{1\frac{1}{2}}$ | 5648.08            | CII               | 3          | $3s^4P_{2\frac{1}{2}}^0 - 3p^4S_{1\frac{1}{2}}$ | 35                  |
| 5640.99          | ScII          | 15         | $a^3P_1 - z^3P_2^0$                             |                    |                   |            |                                                 |                     |
| 5657.88          | ScII          | 25         | $a^3P_2 - z^3P_2^0$                             | 5662.51            | CII               | 4          | $3s^4P_{2\frac{1}{2}}^0 - 3p^4S_{1\frac{1}{2}}$ | 36                  |
| 5658.35          | ScII          | 8          | $a^3P_0 - z^3P_1^0$                             |                    |                   |            |                                                 |                     |
| 5666.64          | NII           | 8          | $3s^3P_1^0 - 3p^3D_2$                           |                    |                   |            |                                                 |                     |
| 5667.16          | ScII          | 10         | $a^3P_1 - z^3P_1^0$                             |                    |                   |            |                                                 |                     |
| 5669.04          | ScII          | 12         | $a^3P_1 - z^3P_0^0$                             |                    |                   |            |                                                 |                     |
| 5669.59          | SiII          | 4          |                                                 |                    |                   |            |                                                 |                     |
| 5676.02          | NII           | 6          | $3s^3P_0^0 - 3p^3D_1$                           | ...                | ...               | ...        | ...                                             | 37                  |
| 5679.56          | NII           | 10         | $3s^3P_2^0 - 3p^3D_3$                           |                    |                   |            |                                                 |                     |
| 5684.20          | ScII          | 15         | $a^3P_2 - z^3P_1^0$                             |                    |                   |            |                                                 |                     |
| 5686.21          | NII           | 6          | $3s^3P_1^0 - 3p^3D_1$                           |                    |                   |            |                                                 |                     |
| 5710.76          | NII           | 6          | $3s^3P_2^0 - 3p^3D_2$                           | 5705.12<br>5707.92 | $\odot +$<br>CrII | ...        | ...                                             | 38                  |
|                  |               |            |                                                 |                    |                   | ...        | $a^4F_{2\frac{1}{2}} - z^6P_{3\frac{1}{2}}^0$   |                     |
| 5747.29          | NII           | 4          | $3s^1P_1^0 - 3p^3D_2$                           | ...                | ...               | ...        | ...                                             | 39                  |
| 5755.0           | [NII]         | ...        | $2p^1D_2 - 2p^1S_0$                             | ...                | ...               | ...        | ...                                             | 40                  |
| 5767.43          | NII           | 3          | $3s^1P_1^0 - 3p^3D_1$                           | ...                | ...               | ...        | ...                                             | 41                  |
| 5830.±           | ...           | ...        | ...                                             | ...                | ...               | ...        | ...                                             | 42                  |
| 5853.69          | BaII          | (200)      | $5^2D_{1\frac{1}{2}} - 6^2P_{1\frac{1}{2}}^0$   | 5867.50            | SiII              | 1          | $4s^4P_{1\frac{1}{2}}^0 - 4p^4P_{\frac{1}{2}}$  | 43                  |
| 5868.40          | SiII          | 3          | $4s^4P_{2\frac{1}{2}}^0 - 4p^4P_{2\frac{1}{2}}$ |                    |                   |            |                                                 |                     |
| 5875.62          | HeI           | 10         | $2^3P_1^0 - 3^3D$                               | ...                | ...               | ...        | ...                                             | 44                  |
| 5889.98          | NaI           | 8          | $3^2S_{\frac{1}{2}} - 3^2P_{1\frac{1}{2}}^0$    | 5889.97            | CII               | 5          | $3d^2D_{2\frac{1}{2}} - 4p^2P_{1\frac{1}{2}}^0$ | 45                  |
| 5895.94          | NaI           | 10         | $3^2S_{\frac{1}{2}} - 3^2P_{\frac{1}{2}}$       | 5891.65            | CII               | 3          | $3d^2D_{1\frac{1}{2}} - 4p^2P_{\frac{1}{2}}^0$  |                     |
| 5927.82          | NII           | 4          | $3p^3P_0 - 3d^3D_{10}$                          | ...                | ...               | ...        | ...                                             | 46                  |
| 5931.79          | NII           | 7          | $3p^3P_1 - 3d^3D_{20}$                          |                    |                   |            |                                                 |                     |
| 5940.25          | NII           | 2          | $3p^3P_1 - 3d^3D_{10}$                          |                    |                   |            |                                                 |                     |
| 5941.67          | NII           | 8          | $3p^3P_2 - 3d^3D_{30}$                          |                    |                   |            |                                                 |                     |
| 5952.39          | NII           | 3          | $3p^3P_2 - 3d^3D_{20}$                          |                    |                   |            |                                                 |                     |
| 5957.61          | SiII          | 5          | $4^2P_{\frac{1}{2}}^0 - 5^2S_{\frac{1}{2}}$     | 5958.53            | OI                | 6          | $3^3P - 5^3D^0$                                 | 47                  |
| 5978.97          | SiII          | 7          | $4^2P_{1\frac{1}{2}}^0 - 5^2S_{\frac{1}{2}}$    |                    |                   |            |                                                 |                     |
| 5991.38          | $\odot +$     | ...        | ...                                             | ...                | ...               | ...        | ...                                             | 48                  |

| (1)<br>$\lambda$                         | (2)<br>Origin               | (3)<br>$I$          | (4)<br>Multiplet                                                                                                       | (5)<br>$\lambda$              | (6)<br>Origin      | (7)<br>$I$       | (8)<br>Multiplet                                                  | (9)<br>Re-<br>marks |
|------------------------------------------|-----------------------------|---------------------|------------------------------------------------------------------------------------------------------------------------|-------------------------------|--------------------|------------------|-------------------------------------------------------------------|---------------------|
| 6010. $\pm$                              | ...                         | ...                 | ...                                                                                                                    | 6013.49                       | TiII(?)            | ...              | ...                                                               | 49                  |
| 6046.04<br>6053.53                       | $\odot +$<br>CrII           | ...<br>2            | ...<br>$b^4F_{3\frac{1}{2}} - z^4P_{2\frac{1}{2}}^0$                                                                   | 6046.34                       | OI                 | 7                | $3^3P - 6^3S_1^0$                                                 | 50                  |
| 6084. $\pm$                              | ...                         | ...                 | ...                                                                                                                    | 6084.12                       | $\odot +$          | ...              | ...                                                               | 51                  |
| 6141.73                                  | BaII                        | (600)               | $5^2D_{2\frac{1}{2}} - 6^2P_{1\frac{1}{2}}^0$                                                                          | ...                           | ...                | ...              | ...                                                               | 52                  |
| 6147.75                                  | FeII                        | 0                   | $b^4D_{1\frac{1}{2}} - z^4P_{\frac{1}{2}}^0$                                                                           | 6149.26                       | FeII               | 00               | $b^4D_{\frac{1}{2}} - z^4P_{\frac{1}{2}}^0$                       | 53                  |
| 6155.15<br>6155.99<br>6156.78<br>6158.20 | $\odot +$<br>OI<br>OI<br>OI | ...<br>7<br>8<br>10 | ...<br>$3^5P_1 - 4^5D^0$<br>$3^5P_2 - 4^5D^0$<br>$3^5P_3 - 4^5D^0$                                                     | ...                           | ...                | ...              | ...                                                               | 54                  |
| 6167.82<br>6173.40                       | NII<br>NII                  | 4<br>3              | $3d^3F_4^0 - 4p^3D_3$<br>$3d^3F_3^0 - 4p^3D_2$                                                                         | ...                           | ...                | ...              | ...                                                               | 55                  |
| 6238.40                                  | FeII                        | 0                   | $b^4D_{1\frac{1}{2}} - z^4P_{1\frac{1}{2}}^0$                                                                          | 6242.52                       | NII                | 5                | $3d^1F_3^0 - 4p^1D_2$                                             | 56                  |
| 6247.57                                  | FeII                        | 1                   | $b^4D_{\frac{1}{2}} - z^4P_{1\frac{1}{2}}^0$                                                                           | 6245.63                       | ScII               | 20               | $a^3P_2 - z^3D_3^0$                                               | 57                  |
| 6284.30                                  | NII                         | 3                   | $3p^1P_1 - 3d^1D_2^0$                                                                                                  | 6278<br>6279.74<br>6283       | ...<br>ScII<br>... | ...<br>15<br>... | ...<br>$a^3P_1 - z^3D_2^0$<br>...                                 | 58                  |
| 6300.23                                  | [OI]                        | 4                   | $2p^3P_2 - 2p^1D_2$                                                                                                    | 6300.70                       | ScII               | 6                | $a^3P_2 - z^3D_2^0$                                               | 59                  |
| 6309.90<br>6320.86                       | ScII<br>ScII                | 15<br>7             | $a^3P_0 - z^3D_1^0$<br>$a^3P_1 - z^3D_1^0$                                                                             | ...                           | ...                | ...              | ...                                                               | 60                  |
| 6340.67<br>6347.10                       | NII<br>SiII                 | 4<br>10             | $3d^3D_3^0 - 4p^3P_2$<br>$4^2S_{\frac{1}{2}} - 4^2P_{1\frac{1}{2}}^0$                                                  | 6332                          | ...                | ...              | ...                                                               | 61                  |
| 6363.88                                  | [OI]                        | 1                   | $2p^3P_1 - 2p^1D_2$                                                                                                    | ...                           | ...                | ...              | ...                                                               | 62                  |
| 6371.36<br>6379.63                       | SiII<br>NII                 | 8<br>5              | $4^2S_{\frac{1}{2}} - 4^2P_{\frac{1}{2}}^0$<br>$3s^3P_1^0 - 3p^1D_2$                                                   | ...                           | ...                | ...              | ...                                                               | 63                  |
| 6416.94                                  | FeII                        | 0                   | $b^4D_{2\frac{1}{2}} - z^4P_{2\frac{1}{2}}^0$                                                                          | ...                           | ...                | ...              | ...                                                               | 64                  |
| 6432.69                                  | FeII                        | ...                 | $a^6S_{2\frac{1}{2}} - z^6D_{2\frac{1}{2}}^0$                                                                          | ...                           | ...                | ...              | ...                                                               | 65                  |
| 6456.40                                  | FeII                        | 3                   | $b^4D_{3\frac{1}{2}} - z^4P_{2\frac{1}{2}}^0$                                                                          | 6453.69<br>6454.55<br>6456.07 | OI<br>OI<br>OI     | 6<br>7<br>9      | $3^5P_1 - 5^5S_2^0$<br>$3^5P_2 - 5^5S_2^0$<br>$3^5P_3 - 5^5S_2^0$ | 66                  |
| 6482.07<br>6491.59<br>6496.92            | NII<br>TiII<br>BaII         | 8<br>2<br>(600)     | $3s^1P_1^0 - 3p^1D_2$<br>$b^2P_{1\frac{1}{2}} - z^2D_{2\frac{1}{2}}^0$<br>$5^2D_{1\frac{1}{2}} - b^2P_{\frac{1}{2}}^0$ | ...                           | ...                | ...              | ...                                                               | 67                  |

| (1)<br>$\lambda$ | (2)<br>Origin | (3)<br>$I$ | (4)<br>Multiplet                                  | (5)<br>$\lambda$ | (6)<br>Origin | (7)<br>$I$ | (8)<br>Multiplet                              | (9)<br>Re-<br>marks |
|------------------|---------------|------------|---------------------------------------------------|------------------|---------------|------------|-----------------------------------------------|---------------------|
| 6516.09          | FeII          | 00         | $a^6S_{2\frac{1}{2}} - z^6D_{3\frac{1}{2}}^0$     | ...              | ...           | ...        | ...                                           | 68                  |
| 6562.82          | Ha            | ...        | $2^2P^0 - 3^2D$                                   | 6548.4           | [NII]         | ...        | $2p^3P_1 - 2p^1D_2$                           | 69                  |
|                  |               |            |                                                   | 6559.59          | TiII          | 1          | $b^2P_1 - z^2D_{1\frac{1}{2}}^0$              |                     |
|                  |               |            |                                                   | 6578.03          | CII           | 10         | $3s^2S_1 - 3p^2P_{1\frac{1}{2}}^0$            |                     |
|                  |               |            |                                                   | 6582.85          | CII           | 8          | $3s^2S_{\frac{1}{2}} - 3p^2P_{\frac{1}{2}}^0$ |                     |
|                  |               |            |                                                   | 6583.9           | [NII]         | ...        | $2p^3P_2 - 2p^1D_2$                           |                     |
| 6604.61          | ScII          | 10         | $b^1D_2 - z^1D_2^0$                               | 6606.99          | TiII          | ...        | $b^2P_{1\frac{1}{2}} - z^2D_{1\frac{1}{2}}^0$ | 70                  |
| 6671.88          | SiII          | 3          | ...                                               | ...              | ...           | ...        | ...                                           | 71                  |
| 6678.15          | HeI           | 6          | $2^1P_1^0 - 3^1D_2$                               |                  |               |            |                                               |                     |
| 6717.3           | [SII]         | ...        | $3p^4S_{1\frac{1}{2}} - 3p^2D_{2\frac{1}{2}}$     | 6721.35          | OII           | 5          | $z^2P_{1\frac{1}{2}} - e^2S_{\frac{1}{2}}^0$  | 72                  |
| 6731.5           | [SII]         | ...        | $3p^4S_{1\frac{1}{2}}^0 - 3p^2D_{1\frac{1}{2}}^0$ | ...              | ...           | ...        | ...                                           | 73                  |
| 6797. $\pm$      | ...           | ...        | ...                                               | 6809.23          | [FeII]        | ...        | $a^4D_{3\frac{1}{2}} - b^4F_{4\frac{1}{2}}$   | 74                  |
| 7043. $\pm$      | ...           | ...        | ...                                               | ...              | ...           | ...        | ...                                           | 75                  |
| 7065.18          | HeI           | 5          | $2^3P_1^0 - 3^3S_1$                               | ...              | ...           | ...        | ...                                           | 76                  |
| 7231.12          | CII           | 6          | $3p^2P_{\frac{1}{2}}^0 - 3d^2D_{1\frac{1}{2}}$    | ...              | ...           | ...        | ...                                           | 77                  |
| 7236.19          | CII           | 8          | $3p^2P_{1\frac{1}{2}} - 3d^2D_{2\frac{1}{2}}$     |                  |               |            |                                               |                     |
| 7281.35          | HeI           | 3          | $2^1P_1^0 - 3^1S^0$                               | ...              | ...           | ...        | ...                                           | 78                  |
| 7308.09          | FeII          | ...        | $b^4D_{1\frac{1}{2}} - z^4D_{1\frac{1}{2}}^0$     | 7310.21          | FeII          | ...        | $b^4D_{\frac{1}{2}} - z^4D_{1\frac{1}{2}}^0$  | 79                  |
| 7318.6           | [OII]         | ...        | $a^2D_{2\frac{1}{2}}^0 - a^2P_{\frac{1}{2}}^0$    | ...              | ...           | ...        | ...                                           | 80                  |
| 7319.4           | [OII]         | ...        | $a^2D_{2\frac{1}{2}}^0 - a^2P_{1\frac{1}{2}}^0$   |                  |               |            |                                               |                     |
| 7320.69          | FeII          | 0          | $b^4D_{2\frac{1}{2}} - z^4D_{1\frac{1}{2}}^0$     |                  |               |            |                                               |                     |
| 7329.9           | [OII]         | ...        | $a^2D_{1\frac{1}{2}}^0 - a^2P_{\frac{1}{2}}^0$    |                  |               |            |                                               |                     |
| 7330.7           | [OII]         | ...        | $a^2D_{1\frac{1}{2}}^0 - a^2P_{1\frac{1}{2}}^0$   |                  |               |            |                                               |                     |

## Remarks

1. Weakening rapidly since beginning of April.
2. These OII-lines first outstanding after the beginning of March.
3. Weakening since end of March.
5. Traced since March 18.
6. Identification with LaII somewhat doubtful, present in  $\alpha$  Persei (*loc. cit.*, p. 25). The NII bands seen since March 16.
7. Seen since March 16.
8. Weakening since end of March.
9. Becoming stronger after March 10, when  $\lambda$  5045 NII also appeared.
12. Probably identical with  $\lambda$  5101.7 in Nova Pictoris (*loc. cit.*, p. 81). Dunham (*loc. cit.*, p. 27) notes: "5100.71. An enhanced line, present in Sirius."
- 13, 14. After January 24 weakened TiII; first traced CII about March 10.

- 15, 16, 19. Weakened after March 22.  
24, 25. Weak after Feb. 17.—31. *FeII* Multipl. : From Dr. Dobbie's unpubl. list.  
33. First traced on 1934 December 27, strong till about March 22, then weaker.  
34. In *a Persei* (*loc. cit.*, p. 32).  
36. Band faded away after end of January. Then *NII* first traced on March 18 ; weakened later ; disappeared between April 3 and April 5.  
37. Strengthened till March 12 ; then stronger, along with the newly appeared *NII*-lines ; faded away between April 3 and April 5.  
38. First traced on December 27.  $\lambda$  5705.12 is in *a Persei* (*loc. cit.*, p. 32) ;  $\lambda$  5707.92 *CrII* is doubtful (*Rowland* ; Miss Moore, pred.).  
38, 39, 41. Strengthened, *NII*, since beginning of March.  
40. First seen weak on December 29, strengthening after January 22, and more from about March 4 till March 20, then weakening till April 1, again strong April 5.  
42, 43. On December 27—(no plate in this region is available from December 22 to December 26)—a band was traced centred at about  $\lambda$  5800 ( $\lambda$  5790– $\lambda$  5810) ; it later weakened and broadened up to about 5860 ; strengthening in February and March, strongest about March 20, then weakening until disappearance after March 25.  
46, 47. Traced December 27 to February 28 ; then appreciably strengthened along with *NII*-bands ; very strong on March 25 and also later.  
48. Unknown origin. Present in *a Persei* (*Dunham, loc. cit.*, p. 33), and in *Nova Pictoris* (*Spencer Jones, loc. cit.*, p. 81, I). Intensity seems appreciably variable. First traced 1934 December 27, weakening till about 1935 January 15, strengthening again till February 3, weakening till February 15, strengthening till March 12, and weakening until disappearance on March 23. On the last suitable plate, April 5, perhaps again a faint trace of it, but uncertain.  
49. Present also in *a Persei* (*loc. cit.*, p. 33). Traced on February 28, strongest on March 12, disappearing after March 20.  
50.  $\lambda$  6046.04 (see also *Dunham, loc. cit.*, p. 33).  
51. Probably identical with  $\lambda$  6087.5 in *Nova Pictoris* (*loc. cit.*, pp. 140, 152), which in the later stages of this nova became the strongest line in its spectrum, but was also measured weak on earlier plates (*loc. cit.*, p. 81, I). In *a Persei* (see *loc. cit.*, p. 33). In *Nova Herculis* weak till 1935 March 20, then strengthened noticeably till March 25, present also later.  
54, 55. After March 12 probably strengthened and broadened under the influence of the *NII* lines at  $\lambda$  6167.82 and  $\lambda$  6173.40. The line at  $\lambda$  6155.15, which seems to be present in this bright band, is the strongest of all unidentified solar lines (see *Rowland*, p. 205). I am much obliged to Prof. H. Dingle for his communication to the effect that its origin is still unknown, and that for a suspected identity with a silicon line the evidence was too scanty. The line may be also noticed in *a Cygni* ( $\lambda$  6156.5 ; *Wright, loc. cit.*, p. 109) ; it is also present in *a Persei* (*loc. cit.*, p. 33).  
56, 57. Strengthened in March, influenced by *NII* 6242.52.  
58.  $\lambda$  6278 and  $\lambda$  6283 are *a Cygni*-lines (*Wright, loc. cit.*, p. 109).  
59. First traced on December 27 ; strengthened towards the end of January, and more in February and March ; then next in brightness to *H $\alpha$*  ; middle of April, when *H $\alpha$*  faded away, strongest line of the spectrum.  
61. Variable intensity. Strong since discovery of the nova till January 25. (Between January 26 and February 1 no plates in this region are available.) From February 2 to February 7 the line was very weak, or not traceable with certainty. February 9 to February 15 : strong again ; February 17 to February 26 : weak again. From March 5 : weak again ; not traceable after April 3.  $\lambda$  6332 is an *a Cygni*-line (*loc. cit.*, p. 109).  
62. First traced after December 16 ; strengthening after December 27 ; behaviour similar to  $\lambda$  6300 [*OI*], to which this line was always next in strength.  
64, 65. Seen till March 16, not traceable later.  
67. Becoming stronger since beginning of March, influenced by the brightening of *NII* 6482.07.

69. After  $H\alpha$  faded away in early April 1935, the forbidden lines at  $\lambda 6548.4$  [NII] and at  $\lambda 6583.9$  [NII] became measurable on plates taken with the Newall refractor after April 11. The last suitable, although underexposed, nova-plate which could be obtained with the Huggins refractor is of April 5 and shows especially, besides the presence of  $H\beta$  and  $H\alpha$ , the strong bands at 6300, 6363, 5755, 5577, and the bands at 4923, 5018, 5169, 5234, 5276, 5284, 5316, 5534, 5875, 5889, 5991 (?), 6516.

74. Identification with the forbidden line  $\lambda 6809.23$  [FeII] somewhat doubtful.

In conclusion I wish to take this opportunity of thanking Professor F. J. M. Stratton sincerely for his very helpful interest in this investigation.

### ON A ZONAL ZERNIKE TEST FOR PARABOLOIDS.

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(Communicated by Prof. F. J. M. Stratton)

*Summary.*—I find that the phase-contrast test of Professor Zernike \* can be used to measure the shapes of paraboloids of low asphericity by testing at the centre of curvature, and taking zonal focal readings in a manner very similar to the zonal knife-edge test. The two methods give results in close agreement.

1. *The Test Procedure.*—An artificial star is placed slightly to one side of the centre of curvature of the mirror, and its image is received on a slide of Zernike disks,† mounted on a focussing carriage provided with three perpendicular micrometer adjustments. The carriage is so aligned that the fringe system given by a centred Z-disk stays central on the mirror as the Z-disk is taken through focus. It is best to select a Z-disk which is known to be suitable for testing a spherical mirror of the same focal ratio as the mirror under test: if such a mirror is not available, select from the Z-disks which give fringe systems of sufficient visibility near paraxial and marginal focus, that one the fringes of which lose visibility most quickly outside the focal range. That is, select the smallest Z-disk giving sufficient contrast for the test.

A black scale, the divisions of which are small teeth, projecting from a narrow strip, is supported across a diameter of the mirror. When a Z-disk is centred, and is sufficiently near mean focus, the scale-teeth appear brightly lit, since they reflect no light to interfere with the supplementary wave created by the presence of the Z-disk at the image. Accordingly one sees

\* F. Zernike, *M.N.*, **94**, 377, 1934, "Diffraction Theory of the Knife-Edge Test, and its improved form, the Phase-Contrast Method"; *Physica*, **1**, No. 8, p. 689, "Beugungstheorie des Schneiden-verfahrens und seiner Verbesserten Form, der Phasenkontrast Methode."

† C. R. Burch, *M.N.*, **94**, 384, 1934, "On the Phase-Contrast Test of F. Zernike."

the luminous scale, cutting across a concentric system of Zernike fringes. The zonal measurement consists in determining the focal settings of the Z-disk which cause the lowest order of interference present to pass through, or midway between, successive pairs of scale teeth. The colour of the ring representing the lowest order of interference changes slightly as its diameter is increased by moving the Z-disk from paraxial to marginal focus, because the order in question changes, so that it is not a case of watching the diametral growth of a ring of given colour, but the growth of a ring surrounded externally and internally by a colour sequence representing increasing orders of interference, *i.e.* increasing apparent lowness of the mirror's surface. The mirror is not diaphragmed for the zonal measurements. The scale is removed while the paraxial setting is made.

2. *The Interpretation of the Test.*—We can regard the Zernike fringes as due to interference between the wave leaving the mirror, and a supplementary wave created by the passage of part of the image light through the Z-disk. If the Z-disk is sufficiently small, the supplementary wave will be substantially spherical over the surface of the mirror, having its centre at the Z-disk. Thus the lowest order of interference present will occur at the zonal radius at which a sphere having its centre at the Z-disk contacts with the wave leaving the mirror. "Rays," reflected at this zonal radius, would pass centrally through the Z-disk, so we can interpret the focal settings of the Z-disk as though they were focal settings of a knife-edge, obtained with only the zone in question exposed. We can only be certain of the sphericity of the supplementary wave, and hence of the validity of this interpretation, if the Z-disk is comparable in diameter with the Airy disk produced by a spherical mirror of the same focal ratio. Such a Z-disk will give fringes heavily diluted with uncoloured light if the asphericity of the paraboloid greatly exceeds half a fringe with respect to the sphere contacting it at the centre, and cutting it at the edge—*i.e.* if the diameter of the paraboloid in cm. exceeds one-twentieth of the cube of its focal ratio. Up to this limit, the dilution is not serious, if the chosen Z-disk is small enough to give a good test on an accurate spherical mirror of the same focal ratio. If the asphericity of the paraboloid greatly exceeds this value, it becomes necessary to use a Z-disk appreciably larger than the Airy disk, in order to obtain sufficient contrast in the fringes, and I would then question the legitimacy of calculating the shape of the mirror to a small fraction of a fringe by treating the zonal readings as ray intersections with the axis.

One can form some idea of the mirror's figure without taking any zonal focal readings. First, the fringe system should be circular, and concentric with the mirror's rim, and it should be possible so to align the focussing carriage that it remains so, at all focal settings: if this is impossible, the mirror is not a centred surface of revolution. Next, when the Z-disk is so set that the highest zone on the mirror (with respect to a reference-sphere imaging at the Z-disk) occurs at  $1/\sqrt{2}$  times the radius of the mirror, the apparent height—and therefore the tint—should be the same at the centre as at the edge. Further, the depth, shown by the difference between central and marginal tints, should be the same for the edge at the paraxial setting,

as it is for the centre at the marginal setting. Finally, the radial width of the ring of colour denoting the highest zone above reference sphere should decrease regularly as the ring is expanded by moving the Z-disk towards marginal focus. Let us suppose a just appreciable change of tint to be caused by a change of height  $\epsilon$  of the mirror's surface. Then if the mirror is paraboloidal, and reference sphere, radius  $\rho$ , contacts it at zonal radius  $\zeta$ , we shall see the changed tint denoting height  $-\epsilon$ , at zonal radius  $z$ , given by

$$(z^2 - \zeta^2)^2 = 8\rho^3\epsilon.$$

Let  $p$  be the zonal radius at which the change of tint is visible when the Z-disk is paraxially set. Then  $p^4 = 8\rho^3\epsilon$ , so that when the Z-disk is set for "contact" at zonal radius  $\zeta$ , we shall see the tint change at the four values of  $z$  given by

$$z^2 = \zeta^2 \pm p^2.$$

It is instructive to make a rough calculation of  $p$ —the radius of that part of the paraboloid which we should not expect to be able to distinguish from a sphere. If we set upper and lower limits for  $\epsilon$  of 0.05 and 0.01 fringe, we find that for  $\rho = 200$  cm.,  $p$  lies between 3 and 2 cm. [I am led to choose these limits by the test recounted in § 4.]

These considerations correspond to the inspection of the "general run" of the shadows given by a knife-edge on the undiaphragmed mirror: they tell us about the shape of the asphericity, but do not give its magnitude accurately, so that they corroborate, but cannot replace, the integration of zonal measurements.

3. *Comparison with the Knife-Edge Test—Example.*—I have said that the mirror is not diaphragmed while the zonal Z-settings are made. It is possible to take zonal knife-edge readings without diaphragms, and indeed it is then easier to repeat the readings consistently. But it is very difficult to be certain of the interpretation of readings so obtained: this has to do with the nature of the knife-edge test. As Lord Rayleigh has pointed out (*Phil. Mag.*, 33, 161, 1917) the influence of the errors of the remoter parts of the mirror can preponderate over that of the nearer parts in determining the shadow intensity given by a centrally set knife-edge. Professor Zernike's analysis of the shadows seen on a unidimensional mirror of asphericity very small compared with the wave-length shows that if the error be expressed as a series of Legendre polynomials, the integrated intensity with which each error is seen is roughly inversely proportional to its order, when the knife-edge is central. Change of focus is represented by a polynomial of the 2nd order, and spherical aberration by one of the 4th order, so that we may expect to underestimate the asphericity of a paraboloid of very low aperture about twice, if we test with a centrally set knife-edge, and no diaphragms. I have seen this effect, on a mirror of about 0.15 fringe asphericity—the contrast for a given knife-edge setting could be reversed, and the apparent spherical aberration doubled, by diaphragming off the outer or inner two-thirds of the mirror.

I also found that when the diaphragm was absent, and the knife-edge so set that the two sides of a given zone began to shadow simultaneously, they

did not continue to shadow equally as the knife-edge was advanced, so that the apparent spherical aberration depended on the depth of shadow for which the setting was made. [The mirror was imaging at  $F/14$ : the pinhole was 0.013 mm. dia.]

I therefore conclude that it is essential to expose only the pair of zones being measured, if one wishes to interpret the knife-edge test accurately. No corresponding restriction holds for the zonal Zernike test, which is, however, subject to the limitation that the asphericity must not be too great.

To compare the results of the two tests, I measured the shape of an un-silvered glass paraboloid by Mr. J. M. Naish, 10 cm. dia., 153.2 cm. radius of curvature, with two Z-disks, and also with a knife-edge, used with diaphragms, taking eight knife-edge settings on each pair of zones. The readings were taken in such a way that I could not know the numerical value of a setting while making it. The zonal errors of image position are given below in thousandths of an inch with respect to that paraboloid which contacts the mirror at the centre, and cuts it at the edge. A + sign denotes that the zone in question focusses short. The pinhole diameter was 0.007 mm.

| Zonal<br>Radius<br>cm. | Z-disk<br>No. 1 | Z-disk<br>No. 2 | Zone<br>cm. | Knife-<br>Edge<br>Mean | Deviations of Individual Knife-Edge<br>Settings |       |       |       |      |      |       |      |
|------------------------|-----------------|-----------------|-------------|------------------------|-------------------------------------------------|-------|-------|-------|------|------|-------|------|
|                        |                 |                 |             |                        |                                                 |       |       |       |      |      |       |      |
| 0                      | +17.9           | +17.0           | 0-1         | +32.1                  | -11.6                                           | -36.6 | +18.7 | +23.1 | -0.3 | -9.6 | +25.4 | -9.4 |
| 1.0                    | +13.0           | +10.6           |             |                        |                                                 |       |       |       |      |      |       |      |
| 1.5                    | +15.2           | +11.8           | 0-2         | +16.1                  | -3.5                                            | -10.4 | +1.2  | +6.9  | -5.3 | +4.9 | -0.5  | +7.0 |
| 2.0                    | +11.6           | +4.3            |             |                        |                                                 |       |       |       |      |      |       |      |
| 2.5                    | +0.3            | +4.1            | 2-3         | -3.9                   | -2.0                                            | -4.7  | +3.2  | +1.2  | +3.4 | +0.3 | +0.8  | -2.6 |
| 3.0                    | -7.4            | -5.4            |             |                        |                                                 |       |       |       |      |      |       |      |
| 3.5                    | -9.6            | -7.5            | 3-4         | -8.8                   | +1.3                                            | +1.9  | +2.6  | +0.1  | -2.4 | -1.5 | +0.8  | -3.0 |
| 4.0                    | -5.5            | -6.9            |             |                        |                                                 |       |       |       |      |      |       |      |
| 4.5                    | +1.2            | +1.5            | 4-5         | +1.2                   | -1.1                                            | +4.8  | +2.0  | -3.9  | -0.5 | +2.8 | -2.7  | -1.4 |

When these data are integrated, we obtain the following values for the error, in hundredths of a fringe ( $\lambda = 5000$  A.U.), the error at the centre and edge being zero by definition.

| Zonal<br>Radius<br>cm. | Z-disk<br>No. 1 | Z-disk<br>No. 2 | Knife-Edge<br>Mean | Knife-Edge<br>$\Delta$ |
|------------------------|-----------------|-----------------|--------------------|------------------------|
| 0                      | 0.0             | 0.0             | 0.0                | 0.0                    |
| 1.0                    | 1.6             | 1.7             | 3.5                | 1.8                    |
| 1.5                    | 3.2             | 3.7             | 6.2                |                        |
| 2.0                    | 4.7             | 6.4             | 7.1                | 2.2                    |
| 2.5                    | 5.8             | 7.7             | 6.4                |                        |
| 3.0                    | 5.8             | 6.8             | 5.1                | 1.2                    |
| 3.5                    | 3.4             | 3.0             | 1.6                |                        |
| 4.0                    | -0.3            | -0.8            | -1.5               | 1.3                    |
| 4.5                    | -0.8            | -1.3            | -1.2               |                        |
| 5.0                    | 0.0             | 0.0             | 0.0                | 2.3                    |

The column headed "Knife-Edge,  $\Delta$ ," gives for each cm. zonal radius the amount by which the knife-edge—mean estimate of the mirror's error must be increased for that radius and all greater radii, if it be supposed that the true knife-edge setting for the zone in question exceeded the mean setting by an amount equal to the mean of the numerical deviations for that zone. It thus gives a measure of the reliability of the knife-edge test. It remained to be seen whether two different observers would obtain closely agreeing asphericities by the zonal Zernike test. Independent settings of Z-disk No. 1 by Mr. Naish and myself differed by a maximum of 0".006 and the resulting values for the mirror's error differed by  $\pm 0.01$  fringe.

I found a greater discrepancy between the two tests with a speculum metal paraboloid 14 cm. dia., 184 cm. radius of curvature, the difference rising to 0.08 fringe when the mean of 8 knife-edge measurements on zones 1 cm. dia. was compared with zonal Zernike measurements every  $\frac{1}{2}$  cm. When I considered only the Z-measurements at 1 cm. intervals, the discrepancy was reduced to 0.05 fringe. This could conceivably be due to the fact that the mirror's figure contains a series of "facets" about  $\frac{1}{2}$  cm. wide. However, this mirror has a badly pitted surface, so that near the paraxial setting, one obtains not a uniform patch of Zernike colour, but a patch composed of a background of blue (high) liberally besprinkled with dots and streaks of yellow (low) around each pit. Thus one can hardly hope to make accurate measurements on it. The asphericity of this mirror is 0.5 fringe: one begins to notice a change of tint—from blue to blue-white—of the colour representing the lowest order of interference, near marginal focus.

I conclude that the zonal Zernike test may be safely used to measure the error of a paraboloid the diameter of which, in cm., does not exceed one-twentieth of the cube of its focal ratio.

One could see the errors of the shape of the asphericity of Mr. Naish's mirror by inspection, without making zonal measurements. The fringe system, when circular, was not quite concentric with the mirror's rim. (This was only obvious near marginal focus.) The mirror thus has very slight irregular coma. Very faint traces of irregularity were visible near the centre, when the Z-disk was paraxially set. These errors were not visible with a knife-edge. When the Z-disk was so set that the zone highest with respect to reference sphere (shown in blue by No. 1 Z-disk) occurred at  $1/\sqrt{2}$  times the mirror's radius, the centre was definitely lower (yellow) than the edge. When the Z-disk was paraxially set, the blue paraxial patch was surrounded by yellow, outside which was a ring of weak reddish brown (lower than yellow), whereas when it was set at marginal focus the centre was yellow. These effects both imply that the central hollow is too steeply bounded for the asphericity to be of paraboloidal type.

The radial width of the blue paraxial patch was 1.5–2 cm.: it decreased to 1–1.5 cm. when the patch was converted into a ring of 3 cm. mean radius by moving the Z-disk towards marginal focus, and then increased to 1.5–2 cm. when its mean radius was 4 cm. The formula given in § 2 would require a radial width of 0.6–1 cm., according as we take 1.5 or 2 cm. for the radial

width of the paraxial patch, so that not only should the ring not have increased in radial width as it approached 4 cm. mean radius, but we would have expected a perceptible decrease. Thus although the estimation of the width of the ring over which the tint is sensibly constant is essentially a rather rough process, one could tell that the turn-down of the outer part of the mirror, with respect to a sphere, was not increasing towards the edge as fast as it ought. Note that the radial width of the paraxial patch is of the expected order of magnitude, the limits given in § 3 becoming, for  $\rho = 153.2$  cm.,  $2.5 - 1.6$  cm.

Since I have discussed the error of Mr. Naish's mirror in some detail, it is just to add (a) a mirror having errors ranging from  $-0.01$  to  $+0.07$  fringe is usually regarded as an exceedingly good mirror; (b) this mirror, which is the first Mr. Naish has made, has less irregular error than any other paraboloid I have seen.

4. *On the Limit of Sensitivity of the Zernike Test.*—To determine the smallest amount of spherical aberration detectable by the Zernike test, I endeavoured to "beat the test," by figuring an accurate spherical mirror, of glass, 15.36 cm. dia., 184.2 cm. radius of curvature. Initially, the mirror was spherical to  $\pm 0.05$  fringe. At the end of three days refiguring, the principal error was prolate spherical aberration, the distances between marginal and paraxial settings of three different Z-disks being  $0''.0057$ ,  $0''.0050$ ,  $0''.0058$ . The mean value,  $0''.0055$ , corresponds to an asphericity with respect to the sphere contacting the mirror at the centre and cutting it at the edge of  $0.031$  fringe. I then stopped figuring, thinking it unlikely that I should improve on this result; moreover, the colours of the Zernike fringes were beginning to show a peculiar effect, indicating that the figure was near to the limit of the test. This effect was more pronounced after the mirror had remained undisturbed for some days, during which time its asphericity decreased appreciably. If we denote the colour sequence, reading from the centre to the edge, when the Z-disk was slightly inside paraxial focus, by A, B, C (blue, deep brown-black, yellow for one Z-disk), the colour sequence just outside marginal focus was C, B, A. I expected, if and when the mirror was perfect, to get at focus the sequence B, B, B. But in fact I got a sequence which can best be described as  $\frac{1}{2}(A+C)$ ,  $(\frac{1}{3}A + \frac{2}{3}B)$ , B. That is, as the Z-disk was taken through focus, the centre changed from blue to yellow without ever becoming brown-black. The disappearance of blue from the intermediate zone was not complete till slightly after the last trace of blue had disappeared from the centre: clearly there was slight prolate spherical aberration. One had the impression that if this trace of prolateness were removed, the colour sequence would be  $\frac{1}{2}(A+C)$ ,  $\frac{1}{3}(A+B+C)$ , B. That is the sort of sequence one would expect from Professor Zernike's analysis, which shows that when a Z-disk of phase retardation either  $\frac{1}{4}$  or  $\frac{3}{4}$  period is placed exactly at focus, the intensity is not uniform over a perfect mirror, but increases towards the centre. The increase can be largely (but not identically) compensated, by placing the Z-disk slightly inside focus if it is retarding  $\frac{1}{4}$  period, and outside if it is retarding  $\frac{3}{4}$  period. My Z-disks, which show blue and yellow as primary

inside and outside focus colours, must be retarding  $m + \frac{1}{4}$  periods for yellow, and  $m + \frac{3}{4}$  periods for blue. Since they cannot be displaced from focus both ways at once, they must always give an increase of blue and/or yellow towards the centre of a perfect mirror. The effect is exaggerated if the Z-disk is large, but even those Z-disks, which produce low fringe visibility when only one fringe off focus, give it appreciably with this mirror. One would expect to get a more delicate test with a Z-disk giving  $\frac{1}{4}$  period retardation for all colours, as the increase of intensity towards the centre would then be automatically compensated in focussing. But since the compensation is not in theory identical over the whole of the mirror's surface, no very great increase in delicacy would seem to be possible. I have not been able to find, on my Z-slides, disks of suitable size which do not give coloured fringes. Z-disks giving blue and yellow as primary colours have, however, the redeeming feature that they give warning, by the increase of central intensity even with the smallest Z-disks giving good fringe visibility, when one is within a reasonable distance of the accuracy at which the simple interpretation of the test becomes open to doubt.

Regarding the errors of this mirror after several days equilibration, marginal and paraxial settings by Mr. S. J. Underhill and myself showed a surprisingly consistent agreement, the marginal-paraxial difference varying from  $0''.0014$  to  $0''.002$ . It seemed to us that the error was not simple spherical aberration, but rather that the marginal setting was correct for the outer half of the mirror, and the paraxial setting was correct for the inner half.  $0''.002$  then implies an asphericity of  $0.017$  fringe with respect to the sphere contacting the mirror at its centre and cutting it at the edge. Faint traces of zonal error, in rings 1 cm. or so wide—due undoubtedly to the facets on the polisher—were visible over the middle half of the mirror when the Z-disk was between marginal and paraxial focus. I do not believe that these zones can be less than  $0.01$  fringe deep.

We inserted a figured "error-plate"—a plane parallel plate on which I had worked a zone  $0.35$  fringe low—in the line of sight, so as to produce on the mirror two artificial zones each  $0.09$  fringe high, the one being impressed on the wave on its way to the mirror, and the other, on a different part of the wave, on its way back from the mirror. We could then see the two zones as deep brown-black rings, black-blue where they intersected, the rest of the mirror being a dark yellow. The residual error of the mirror itself could no longer be distinguished.

I conclude that  $0.017$  fringe is about the smallest amount of spherical aberration that can be detected with reasonable certainty by the Zernike test, in the form in which I use it. This is in fact the amount predicted by Professor Zernike from theoretical considerations.

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1935 March 4.

# RADIAL VELOCITIES OF GLOBULAR CLUSTERS AND NEBULÆ.

*S. W. Shiveshwarkar, B.A.*

1. The analysis of the radial velocities of Globular Clusters and the extra-galactic nebulæ has already been done by several writers, notably by Gustaf Strömberg,\* ten Bruggencate,† and Pahlen and Freundlich.‡

In the present paper the radial velocities of the 18 Globular Clusters used by the above writers, together with three additional Globular Clusters, are analysed. Table I contains a list of these 21 clusters with their numbers in Dreyer's New General Catalogue, their galactic longitudes  $l$  and latitudes  $b$  and their radial velocities  $R$ . The pole of the galactic equator is taken to be

$$R.A. = 18^h 40^m, \quad Dec. = +28^\circ,$$

the galactic longitude  $l$  being measured from the node of the galactic equator on the celestial equator. The data for the first 18 objects in Table I have been taken from the *Lick Publications*, vol. xviii, while the data for the last three has been taken from *P.A.S.P.*, 1930.

2. The elements of the Solar Motion with regard to the Globular Clusters were first determined using the method of Least Squares assuming that their peculiar radial velocities can be treated as haphazard errors obeying the normal law of distribution. The work was done in galactic co-ordinates ( $\xi, \eta, \zeta$ ), the galactic frame of reference being chosen as above, the  $\xi$ -axis being taken at  $l=0, b=0$ . The equation of condition used was

$$a\xi + b\eta + c\zeta + R + K = 0, \quad (1)$$

including a  $K$ -term. Here

$$a = \cos l \cos b; \quad b = \sin l \cos b; \quad c = \sin b, \quad (2)$$

and ( $\xi, \eta, \zeta$ ) are the components of solar motion. The weights attached to the observations are given in Table I, these being set according to either the uncertainty of the observation or the observation being the mean of several observations.

The solutions were obtained without and with the  $K$ -term. The  $K$ -term came out so unexpectedly small as to make the two solutions almost identical. The solutions with their probable errors are :

| Without $K$ -term    | With $K$ -term       | (2) |
|----------------------|----------------------|-----|
| $\xi = +123 \pm 25$  | $\xi = +121 \pm 38$  |     |
| $\eta = +285 \pm 45$ | $\eta = +288 \pm 48$ |     |
| $\zeta = +63 \pm 32$ | $\zeta = +54 \pm 32$ |     |
| $K = 0$              | $K = 0.4 \pm 29$     |     |

\* *Ap. J.*, **61**, 353, 1925.

† *P.N.A.S.*, **16**, 111, 1930.

‡ *Zeits. f. Astroph.*, **1**, 200, 1930.

Strömberg \* gives the solar motion with regard to the first 18 Globular Clusters in Table I as 329 km./sec. towards the point :

$$\text{R.A.} = 21^{\text{h}} 20^{\text{m}}, \quad \text{Dec.} = +65^{\circ}.$$

In galactic co-ordinates this becomes

$$\xi = +105; \quad \eta = +303; \quad \zeta = +61,$$

which agrees fairly well with the above solution with regard to the 21 Globular Clusters in Table I.

TABLE I

| N.G.C. | <i>l</i> | <i>b</i> | <i>R</i><br>(km./sec.) | Weight | Residuals        |               |
|--------|----------|----------|------------------------|--------|------------------|---------------|
|        |          |          |                        |        | Without <i>K</i> | With <i>K</i> |
| 1851   | 212      | -34.5    | +315                   | 1      | +69              | +74           |
| 1904   | 194      | -28      | +235                   | 1      | +39              | +45           |
| 5024   | 305      | +79      | -185                   | 1.5    | -154             | -163          |
| 5272   | 8        | +77.5    | -132                   | 1.5    | -36              | -42           |
| 5904   | 332      | +46      | +10                    | 1      | +38              | +30           |
| 6093   | 320.5    | +18      | +70                    | 1      | +7               | +2            |
| 6205   | 27       | +48      | -243                   | 2      | -19              | -25           |
| 6218   | 344      | +25      | +160                   | 1      | +223             | +217          |
| 6229   | 40       | +40      | -100                   | 0.5    | +154             | +148          |
| 6266   | 322      | +7       | +50                    | 1      | -20              | -24           |
| 6273   | 324.5    | +9       | +30                    | 1      | -25              | -29           |
| 6333   | 333      | +10      | +225                   | 1      | +217             | +212          |
| 6341   | 36       | +35      | -160                   | 1      | +95              | +90           |
| 6626   | 335      | -7       | 0                      | 1      | -17              | -18           |
| 6934   | 20       | -20      | -350                   | 0.5    | -172             | -168          |
| 7078   | 33       | -28      | -100                   | 1      | +99              | +103          |
| 7089   | 21       | -36      | -10                    | 1      | +128             | +133          |
| 7099   | 355      | -48.5    | -125                   | 1      | -108             | -102          |
| 6440   | 335      | +2       | -175                   | 1      | -182             | -185          |
| 6981   | 3        | -34      | -180                   | 1      | -101             | -97           |
| 7006   | 32       | -21      | -350                   | 1      | -134             | -130          |

ten Bruggencate,† using an equation of condition which takes into account the effect of the “red-shift,” finds the Solar Motion. But his solution does not agree with the above at all. The inclusion of the “red-shift” term seems to be doubtful. His probable errors are also comparatively large.

3. The velocity of the mean of the local stars with regard to the centre of the Galaxy was determined. This centre was taken to be the mean of the Globular Clusters. Its galactic longitude was taken to be  $325^{\circ}$  according to the determination by Shapley.‡ The Solar Motion with regard to the local stars was taken to be 19.5 km./sec. towards the apex :

$$\text{R.A.} = 18^{\text{h}}, \quad \text{Dec.} = +30^{\circ}.$$

The galactic components ( $\xi'$ ,  $\eta'$ ,  $\zeta'$ ) of this velocity are :

$$\xi' = +16, \quad \eta' = +8, \quad \zeta' = +8.$$

\* *Ap. J.*, 61, 353, 1925.

† *P.N.A.S.*, 16, 111, 1930.

‡ *Star Clusters*, p. 22, 1930.

The radial component  $II_0$ , the clock-wise transverse components  $\Theta_0$  and  $Z_0$ , the component perpendicular to the plane of the Milky Way, are given below.  $\Theta_0$  is, of course, the galactic rotation in the vicinity of the Sun.

| Without $K$               |  | With $K$                  |     |
|---------------------------|--|---------------------------|-----|
| $II_0 = + 81 \pm 32$      |  | $II_0 = + 82 \pm 44$      | (3) |
| $\Theta_0 = + 286 \pm 41$ |  | $\Theta_0 = + 287 \pm 39$ |     |
| $Z_0 = + 55 \pm 32$       |  | $Z_0 = + 46 \pm 32$       |     |
| $K = 0$                   |  | $K = 0.4 \pm 29$          |     |

The transformation from the components  $(\xi, \eta, \zeta)$  to the components  $(II, \Theta, Z)$  is

$$\left. \begin{aligned} \xi &= \Theta \cos \theta - II \sin \theta \\ \eta &= \Theta \sin \theta + II \cos \theta \\ \zeta &= Z \end{aligned} \right\}, \tag{4}$$

where  $\theta = 55^\circ$ .

To obtain the weights and probable errors of  $II, \Theta, Z$  we must construct the normal equations for these components. This cannot be done simply by substituting (4) in the normal equations obtained from (1).<sup>\*</sup> For (4) transforms (1) into the new equation of condition

$$(a \cos \theta + b \sin \theta)\Theta + (-a \sin \theta + b \cos \theta)II + cZ + R + K = 0, \tag{5}$$

or

$$a'\Theta + b'II + cZ + R + K = 0 \quad (\text{say})$$

and  $[a'a'] = [aa] \cos^2 \theta + 2[ab] \cos \theta \sin \theta + [bb] \sin^2 \theta$ , etc., from which it is easy to calculate the normal equations for  $(\Theta, II, Z)$  from those for  $(\xi, \eta, \zeta)$ .

4. A list of extra-galactic nebulae is given in Table II with their numbers in N.G.C., their galactic longitudes  $l$  and latitudes  $b$  and their radial velocities  $R$ .

TABLE II

| N.G.C. | $l$      | $b$      | $R$  | Residual<br>radial<br>velocity $R'$ |
|--------|----------|----------|------|-------------------------------------|
|        | $^\circ$ | $^\circ$ |      |                                     |
| 205    | 89       | -21      | -300 | - 55                                |
| 221    | 89       | -22      | -210 | + 32                                |
| 224    | 90       | -21      | -270 | - 26                                |
| 404    | 95.5     | -27      | - 25 | +189                                |
| 598    | 102      | -31      | -285 | -100                                |
| 1023   | 113      | -19      | +300 | +482                                |
| 3031   | 109      | +42      | - 30 | +182                                |
| 3034   | 108      | +41      | +290 | +507                                |
| 4214   | 121      | +79      | +300 | +567                                |
| 4449   | 100      | +73      | +200 | +336                                |
| 4736   | 84       | +76      | +290 | +423                                |
| 4826   | 295      | +83      | +150 | +187                                |
| 5194   | 70       | +68      | +270 | +441                                |
| 5195   | 70       | +68      | +240 | +418                                |
| 5457   | 67       | +59      | +300 | +514                                |
| 6822   | 353      | -20      | -150 | - 90                                |

<sup>\*</sup> I am indebted to Sir Arthur Eddington for bringing this point to my notice.

Only those nebulae are selected whose observed radial velocities are not greater than +300 km./sec. The last column gives their residual radial velocities  $R'$ , these being obtained from

$$R' = a\xi + b\eta + c\zeta + R \quad (6)$$

where

$$a = \cos l \cos b, \quad b = \sin l \cos b, \quad c = \sin b,$$

and  $(\xi, \eta, \zeta)$  are the components of solar motion given in (2).

The positive velocities remain positive and increase in magnitude, while the negative velocities either change their sign or, if they remain negative, are considerably reduced in magnitude. In (6), only the solution without the  $K$ -term was used, as the other solution will not make much difference, the  $K$ -term being exceedingly small.

I am indebted to Dr. Smart for valuable help and guidance.

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*Cambridge :*  
1935 March 25.

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#### *Erratum.*

*M.N.*, 95, 383, 1935, *Council Note on Minor Planets.*

Professor L. Volta points out that the identification of Planet 1934 AA (discovered at the Observatory of Pino Torinese, 1934 January) with Planet 3111 is erroneous.